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An Identification Problem of Coefficient in the Special Form at Source Function for Multi-dimensional Parabolic Equation with Cauchy Data

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The existence and uniqueness of solution of the identification problem for multi-dimensional parabolic equation with source function of the special form in the case of Cauchy's data has been proved in this paper.

Keywords: inverse problem, identification of coefficient at source function, multi-dimensional parabolic equation, method of weak approximation, existence and uniqueness of solution, Cauchy problem.

The identification problem of source function of the special form in multi-dimensional parabolic equation with Cauchy's data was considered in this paper. Unknown coefficient depending on the all variables has form of the sum of functions each of which depends on time and a spatial variable.

Using overdetermination conditions the inverse problem is reduced to direct problem for loaded parabolic equation. Solvability of the direct problem is researched by means of the method of weak approximation [1–3]. Theorems of existence and uniqueness of solution of the original inverse problem have been proved in classes of smooth bounded functions.

Correctness of the identification problem of source function in two-dimensional parabolic equation, where unknown coefficient has form of the sum of two functions, has been investigated earlier in [4]. Behaviour of solution of the two-dimensional identification problem of source function, when time goes to infinity, has been researched in [5]. In [6] the case, when unknown coefficient has form of the product of two functions, has been studied. The identification problem of source function depending on (t, x) has been investigated in [7]. In [8,9] identification problems of certain coefficients at higher-order derivatives for parabolic equations with input data in the special form have been studied. Initial-boundary value problems of identification of coefficient at source function have been considered in [10, 11].

1. The problem formulation

In the domain $G_{[0,T]} = \left\{(t, x) \,|\, 0 \leqslant t \leqslant T,\ x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n\right\}$ we consider the Cauchy problem for the following parabolic equation

$$u_t = L\left(u(t, x)\right) + f(t, x) \cdot \lambda(t, x),\ t \in (0, T),\ x \in \mathbb{R}^n, \tag{1}$$

$$L\left(u(t, x)\right) = \sum_{k=1}^{n} b_k(t) u_{x_k x_k}(t, x) + \sum_{k=1}^{n} c_k(t) u_{x_k}(t, x), \tag{2}$$

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with the initial data

$$u(0,x) = u_0(x),\ x \in \mathbb{R}^n. \tag{3}$$

Here coefficients $b_k(t) \geqslant b_0 > 0$, $c_k(t)$, $k = 1,2,\dots,n$, are real-valued, continuous and bounded in $[0,T]$ functions.

In the problem it is required to determine the solution of equation (1) $u(t,x)$ and also coefficient, which is introduced as

$$\lambda(t,x) = \sum_{k=1}^{n} \lambda_k(t, x_k). \tag{4}$$

Let function $u(t,x)$ satisfy conditions of overdetermination:

$$u(t, a^1_{k-1}, x_k, a^2_{k+1}) = \varphi_k(t, x_k),\ k = 1,2,\dots,n, \tag{5}$$

here vectors $a^1_k, a^2_k,\ k = 1,2,\dots,n$, are:

$$a_0 = (\alpha_1, \alpha_2, \dots, \alpha_n),\ a^1_k = (\alpha_1, \alpha_2, \dots, \alpha_k),\ a^2_k = (\alpha_k, \alpha_{k+1}, \dots, \alpha_n),$$

$$\alpha_k = \text{const},\ k = 1,2,\dots,n.$$

Suppose that the following conditions

$$u_0(a^1_{k-1}, x_k, a^2_{k+1}) = \varphi_k(0, x_k),\ k = 1,2,\dots,n, \tag{6}$$

$$\varphi_1(t, \alpha_1) = \varphi_2(t, \alpha_2) = \dots = \varphi_n(t, \alpha_n), \tag{7}$$

and conditions on function $f(t,x)$:

$$\left| f(t, a^1_{k-1}, x_k, a^2_{k+1}) \right| \geqslant \delta_k > 0,\ \delta_k = \text{const},\ k = 1,2,\dots,n, \tag{8}$$

are hold.

We will use the following notation:

$$D^s v(x) = D^{(s_1, s_2, \dots, s_n)} v(x) = \frac{\partial^{|s|} v(x_1, x_2, \dots, x_n)}{\partial x_1^{s_1} \partial x_2^{s_2} \dots \partial x_n^{s_n}},$$

where $s = (s_1, s_2, \dots, s_n)$ is multi-index, $s_r \geqslant 0$ are integer, $r = 1,2,\dots,n$, $|s| = s_1 + \ldots + s_n$.

Let functions $f(t,x)$, $u_0(x)$, $\varphi_k(t,x_k), k = 1,2,\dots,n$, given in the domains $G_{[0,T]}$, $\mathbb{R}^n$, $[0,T] \times \mathbb{R}^1$ accordingly, be sufficiently smooth, have the all continuous derivatives, which satisfy the inequalities (9)–(11).

$$\left| D^s f(t,x) \right| \leqslant C,\ s = (s_1, \dots, s_n),\ s_r = 0,1,\dots,6,\ r = 1,\dots,n, \tag{9}$$

$$\left| D^s u_0(x) \right| \leqslant C,\ s = (s_1, \dots, s_n),\ s_r = 0,1,\dots,6,\ r = 1,\dots,n, \tag{10}$$

$$\left| \frac{\partial^{i+j}}{\partial t^i \partial x_k^j} \varphi_k(t, x_k) \right| \leqslant C,\ i = 0,1; j = 0,1,\dots,6;\ k = 1,2,\dots,n. \tag{11}$$

2. Reducing the inverse problem to the direct problem

We reduce the inverse problem (1)–(5) to the auxiliary direct problem. Substituting $x = a_0$, $x_k = \alpha_k, k = 1, \ldots, n$, into (1) we obtain the formulas of coefficient at source function:

$$\lambda_1(t,\alpha_1) + \lambda_2(t,\alpha_2) + \ldots + \lambda_n(t,\alpha_n) = \frac{u_t(t,a_0) - L\left(u(t,a_0)\right)}{f(t,a_0)}, \tag{12}$$

$$\begin{cases} \lambda_1(t,x_1) + \lambda_2(t,\alpha_2) + \ldots + \lambda_n(t,\alpha_n) = \dfrac{u_t(t,x_1,a_2^2) - L\left(u(t,x_1,a_2^2)\right)}{f(t,x_1,a_2^2)}, \\ \lambda_1(t,\alpha_1) + \lambda_2(t,x_2) + \ldots + \lambda_n(t,\alpha_n) = \dfrac{u_t(t,\alpha_1,x_2,a_3^2) - L\left(u(t,\alpha_1,x_2,a_3^2)\right)}{f(t,\alpha_1,x_2,a_3^2)}, \\ \ldots \\ \lambda_1(t,\alpha_1) + \ldots + \lambda_{n-1}(t,\alpha_{n-1}) + \lambda_n(t,x_n) = \dfrac{u_t(t,a_{n-1}^1,x_n) - L\left(u(t,a_{n-1}^1,x_n)\right)}{f(t,a_{n-1}^1,x_n)}. \end{cases} \tag{13}$$

We sum up of the equations (13) and substitute the right part of equation (12) into the obtained formula.

$$\sum_{k=1}^{n} \lambda_k(t,x_k) = \frac{u_t(t,x_1,a_2^2) - L\left(u(t,x_1,a_2^2)\right)}{f(t,x_1,a_2^2)} + \cdots + \frac{u_t(t,a_{n-1}^1,x_n) - L\left(u(t,a_{n-1}^1,x_n)\right)}{f(t,a_{n-1}^1,x_n)} - \\ - (n-1)\cdot\frac{u_t(t,a_0) - L\left(u(t,a_0)\right)}{f(t,a_0)}.$$

From conditions (5) it follows that $u_{x_kx_k}(t,a_{k-1}^1,x_k,a_{k+1}^2) = \dfrac{\partial^2}{\partial x_k^2}\varphi_k(t,x_k)$, $k = 1,2,\ldots,n$. Therefore we get the formula of unknown coefficient in the form

$$\lambda(t,x) = \sum_{k=1}^{n} \lambda_k(t,x_k) = g(t,x) - \\ - \sum_{i=1}^{n} \frac{\sum\limits_{\substack{k=1,\\k\neq i}}^{n} b_k(t) u_{x_kx_k}(t,a_{i-1}^1,x_i,a_{i+1}^2) + \sum\limits_{\substack{k=1,\\k\neq i}}^{n} c_k(t) u_{x_k}(t,a_{i-1}^1,x_i,a_{i+1}^2)}{f(t,a_{i-1}^1,x_i,a_{i+1}^2)}. \tag{14}$$

Here function $g(t,x)$ is known and has the form

$$g(t,x) = \sum_{k=1}^{n} \frac{\frac{\partial}{\partial t}\varphi_k(t,x_k) - b_k(t)\frac{\partial^2}{\partial x_k^2}\varphi_k(t,x_k) - c_k(t)\frac{\partial}{\partial x_k}\varphi_k(t,x_k)}{f(t,a_{k-1}^1,x_k,a_{k+1}^2)} - \\ - (n-1)\cdot\frac{\frac{\partial}{\partial t}\varphi_1(t,\alpha_1) - \sum\limits_{k=1}^{n}\Big(b_k(t)\frac{\partial^2}{\partial x_k^2}\varphi_k(t,\alpha_k) + c_k(t)\frac{\partial}{\partial x_k}\varphi_k(t,\alpha_k)\Big)}{f(t,a_0)}.$$

Let us introduce the functions:

$$\Phi_0(t,x) = -\frac{f(t,x)}{f(t,a_0)}, \\ \Phi_k(t,x) = -\frac{f(t,x)}{f(t,a_{k-1}^1,x_k,a_{k+1}^2)}, \; k = 1,2,\ldots,n, \tag{15}$$

$$G(t,x) = f(t,x) \cdot g(t,x). \tag{16}$$

Substituting (14) into (1) and using (15) and (16) we obtain the direct problem for the equation

$$u_t = L\left(u(t,x)\right) + \sum_{i=1}^{n} \Phi_i(t,x) \cdot \left(\sum_{\substack{k=1,\\ k\neq i}}^{n} b_k(t) u_{x_k x_k}(t, a^1_{i-1}, x_i, a^2_{i+1}) + \right.$$
$$\left. + \sum_{\substack{k=1,\\ k\neq i}}^{n} c_k(t) u_{x_k}(t, a^1_{i-1}, x_i, a^2_{i+1}) \right) + G(t,x), \tag{17}$$

with initial data (3).

3. Solvability of the direct problem

To prove solvability of the direct problem we will use the method of weak approximation. We split equation (17) into three fractional steps and make time shift to $\frac{\tau}{3}$ in traces of unknown function.

$$u^\tau_t = 3 \cdot L\left(u^\tau(t,x)\right), \; j\tau < t \leqslant \left(j + \tfrac{1}{3}\right)\tau; \tag{18}$$

$$u^\tau_t = 3 \cdot \sum_{i=1}^{n} \Phi_i(t,x) \cdot \left(\sum_{\substack{k=1,\\ k\neq i}}^{n} b_k(t) u^\tau_{x_k x_k}(t - \frac{\tau}{3}, a^1_{i-1}, x_i, a^2_{i+1}) + \right.$$
$$\left. + \sum_{\substack{k=1,\\ k\neq i}}^{n} c_k(t) u^\tau_{x_k}(t - \frac{\tau}{3}, a^1_{i-1}, x_i, a^2_{i+1}) \right), \; \left(j + \tfrac{1}{3}\right)\tau < t \leqslant \left(j + \tfrac{2}{3}\right)\tau; \tag{19}$$

$$u^\tau_t = 3 \cdot G(t,x), \; \left(j + \tfrac{2}{3}\right)\tau < t \leqslant (j+1)\tau, \tag{20}$$

$$u^\tau(0,x) = u_0(x), \tag{21}$$

$j = 0, 1, \ldots, (N-1)$, $N\tau = T$.

Below j-th whole step means interval $\big(j\tau, (j+1)\tau\big]$ and r-th fractional step of j-th whole step means interval $\Big(\big(j + \frac{r-1}{3}\big)\tau, \; \big(j + \frac{r}{3}\big)\tau\Big]$.

Let us introduce the following notations:

$$\begin{aligned}
&U_s(0) = \sup_{x\in\mathbb{R}^n} \left|D^s u_0(x)\right|,\\
&U^\tau_s(t) = \sup_{j\tau<\xi\leq t} \sup_{x\in\mathbb{R}^n} \left|D^s u^\tau(\xi,x)\right|, \; j\tau < t \leq (j+1)\tau,\\
&s = (s_1, \ldots, s_n), \; s_r = 0, 1, \ldots, 6, \; r = 1, \ldots, n,\\
&U(0) = \sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,\ldots,6,\\ r=1,\ldots,n}} U_s(0), \qquad U^\tau(t) = \sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,\ldots,6,\\ r=1,\ldots,n}} U^\tau_s(t).
\end{aligned} \tag{22}$$

The following statements are valid

1. $\left|D^s u^\tau(\xi,x)\right| \leqslant U^\tau_s(t) \leqslant U^\tau(t), \; j\tau < \xi \leq t \leq (j+1)\tau;$ (23)

2. Functions $U_s^\tau(t), U^\tau(t)$ are nonnegative and nondecreasing in every time step $(j\tau, (j+1)\tau]$.

Let us prove a priori estimations, which guarantee the compactness of family of solutions $u^\tau(t,x)$ of the problem (18)–(21).

Zeroth whole step ($j=0$) is considered. In first fractional step, $t \in \left(0, \frac{\tau}{3}\right]$, we deal with the equation

$$u_t^\tau = 3 \cdot L\left(u^\tau(t,x)\right).$$

Using the principle of maximum for Cauchy problem we have

$$\left|u^\tau(\xi,x)\right| \leqslant \sup_{x\in\mathbb{R}^n}\left|u_0(x)\right|, \, 0<\xi\leqslant t, \, t\in\left(0,\tfrac{\tau}{3}\right]. \tag{24}$$

After applying the differential operator D^s with $s=(s_1,\ldots,s_n)$, $s_r = 0,1,\ldots,6$, $r=1,\ldots,n$, to the equation (18) and initial condition (21), using the principle of maximum, we obtain

$$\left|D^s u^\tau(\xi,x)\right| \leqslant \sup_{x\in\mathbb{R}^n}\left|D^s u_0(x)\right|, \; 0<\xi\leqslant t, \; t\in\left(0,\tfrac{\tau}{3}\right]. \tag{25}$$

We apply $\sup\limits_{x\in\mathbb{R}^n}$ and $\sup\limits_{0<\xi\leqslant t}$ to both parts of inequalities (24), (25) and sum up the results. In terms of U^τ (22) we get the following estimation

$$U^\tau(t) \leqslant U(0), \; 0<t\leqslant \frac{\tau}{3}. \tag{26}$$

Let us consider the second fractional step of zeroth whole step, $t\in\left(\frac{\tau}{3}, \frac{2\tau}{3}\right]$.

$$u_t^\tau = 3\cdot\sum_{i=1}^n \Phi_i(t,x)\cdot\left(\sum_{\substack{k=1,\\k\neq i}}^n b_k(t)u^\tau_{x_kx_k}(t-\tfrac{\tau}{3}, a^1_{i-1}, x_i, a^2_{i+1}) + \sum_{\substack{k=1,\\k\neq i}}^n c_k(t)u^\tau_{x_k}(t-\tfrac{\tau}{3}, a^1_{i-1}, x_i, a^2_{i+1})\right).$$

Integrating equation (19) we have

$$u^\tau(\xi,x) = u^\tau\left(\tfrac{\tau}{3},x\right) + 3\sum_{i=1}^n\int_{\frac{\tau}{3}}^{\xi}\Phi_i(\theta,x)\cdot\left(\sum_{\substack{k=1,\\k\neq i}}^n b_k(\theta)u^\tau_{x_kx_k}(\theta-\tfrac{\tau}{3}, a^1_{i-1}, x_i, a^2_{i+1}) + \right.$$
$$\left. + \sum_{\substack{k=1,\\k\neq i}}^n c_k(\theta)u^\tau_{x_k}(\theta-\tfrac{\tau}{3}, a^1_{i-1}, x_i, a^2_{i+1})\right)d\theta, \; \tfrac{\tau}{3}<\xi\leqslant t, \; t\in\left(\tfrac{\tau}{3}, \tfrac{2\tau}{3}\right].$$

From (8), (9), (15) it follows that functions $\Phi_k(t,x)$ and their derivatives $D^s\Phi_k(t,x)$, $k=1,\ldots,n$, $s=(s_1,\ldots,s_n)$, $s_r=0,1,\ldots,6$, $r=1,\ldots,n$, are bounded. Therefore the following inequality is valid

$$\sup_{0<\xi\leqslant t}\sup_{x\in\mathbb{R}^n}\left|u^\tau(\xi,x)\right| \leqslant \sup_{x\in\mathbb{R}^n}\left|u^\tau\left(\tfrac{\tau}{3},x\right)\right| +$$
$$+ C\cdot\int_{\frac{\tau}{3}}^t\left(\sum_{k=1}^n\sup_{0<\theta\leqslant t}\sup_{x\in\mathbb{R}^n}\left|u^\tau_{x_kx_k}(\theta-\tfrac{\tau}{3},x)\right| + \sum_{k=1}^n\sup_{0<\theta\leqslant t}\sup_{x\in\mathbb{R}^n}\left|u^\tau_{x_k}(\theta-\tfrac{\tau}{3},x)\right|\right)d\theta.$$

Here and further we suppose, that $C>1$ are different constants, which depend on the initial data and do not depend on parameter τ.

We apply differential operator D^s with $s = (s_1, \ldots, s_n)$, $s_r = 0, 1, \ldots, 6$, $r = 1, \ldots, n$, to (19) and integrate the obtained equation. Taking into account (22), (23), (26) we get

$$U^\tau(t) \leqslant U(0) + C \cdot \int_{\frac{\tau}{3}}^{\frac{2\tau}{3}} U^\tau(\theta - \tfrac{\tau}{3})\, d\theta,\ t \in \left(\tfrac{\tau}{3}, \tfrac{2\tau}{3}\right]. \tag{27}$$

In the third fractional step of zeroth whole step, $t \in \left(\frac{2\tau}{3}, \tau\right]$, we consider the equation

$$u_t^\tau = 3 \cdot G(t, x).$$

Similarly to second fractional step using (9)–(11), (16), (22), (23), (27) we have

$$U^\tau(t) \leqslant U(0) + C \int_{\frac{\tau}{3}}^{\tau} \left(U^\tau\left(\theta - \tfrac{\tau}{3}\right) + 1 \right) d\theta,\ t \in \left(\tfrac{\tau}{3}, \tau\right].$$

With properties of definite integral, taking into consideration that function $U^\tau(t)$ is nondecreasing we obtain

$$U^\tau(t) \leqslant U(0) + C \int_0^{\tau} \left(U^\tau(\theta) + 1 \right) d\theta,\ t \in (0, \tau].$$

Next under Gronwall's lemma the following estimations are valid.

$$U^\tau(t) \leqslant U(0) \cdot e^{C\tau} + \frac{C}{C} \cdot \left(e^{C\tau} - 1\right),$$

$$U^\tau(t) \leqslant (U(0) + 1) \cdot e^{C\tau} - 1,\ \forall t \in (0, \tau]. \tag{28}$$

Then we consider the first whole time step, $t \in (\tau, 2\tau]$. Here similarly to zeroth whole step we have

$$U^\tau(t) \leqslant (U(\tau) + 1) \cdot e^{C\tau} - 1 \leqslant \left[\text{under (28)}\right] \leqslant$$
$$\leqslant (U(0) + 1) \cdot e^{C\tau} \cdot e^{C\tau} - 1 = (U(0) + 1) \cdot e^{2C\tau} - 1,$$

that is

$$U^\tau(t) \leqslant (U(0) + 1) \cdot e^{2C\tau} - 1,\ \forall t \in (0, 2\tau].$$

In the second whole time step, where $t \in (2\tau, 3\tau]$, we get the estimation

$$U^\tau(t) \leqslant (U(0) + 1) \cdot e^{3C\tau} - 1,\ \forall t \in (0, 3\tau].$$

Through finite number of steps in interval $\left((N-1)\tau, N\tau\right]$ we obtain

$$U^\tau(t) \leqslant (U(0) + 1) \cdot e^{CN\tau} - 1 = (U(0) + 1) \cdot e^{CT} - 1 \leqslant C,\ \forall t \in \left((N-1)\tau, N\tau\right].$$

Finally the estimation is hold:

$$U^\tau(t) \leqslant (U(0) + 1) \cdot e^{CT} - 1 \leqslant C,\ \forall t \in [0, T].$$

There by the uniform on τ estimations are proved:

$$\left| D^s u^\tau(t, x) \right| \leqslant C,\ s = (s_1, \ldots, s_n),\ s_r = 0, 1, \ldots, 6,\ r = 1, \ldots, n,\ (t, x) \in G_{[0,T]}. \tag{29}$$

From estimations (29) it follows that the right parts of equations (18)–(20) are bounded uniformly on τ in every time step, therefore the left parts of the equations are bounded uniformly on τ too.

$$\left| u_t^\tau(t, x) \right| \leqslant C,\ (t, x) \in G_{[0,T]}.$$

Applying differential operator D^s with $s = (s_1, \ldots, s_n)$, $s_r = 0, 1, \ldots, 4$, $r = 1, \ldots, n$, to (18)–(20) we obtain the following estimations

$$\left|D^s u_t^\tau(t,x)\right| \leqslant C,\ s = (s_1, \ldots, s_n),\ s_r = 0,1,\ldots,4,\ r = 1,\ldots,n,\ (t,x) \in G_{[0,T]}. \tag{30}$$

Estimations (29), (30) guarantee that conditions of Arzela's theorem about compactness are fulfilled. With the help of the Arzela's theorem the subsequence $u^{\tau_k}(t,x)$ of sequence $u^\tau(t,x)$ of solutions of the problem (18)–(21) with derivatives $D^s u^\tau$, $s = (s_1, \ldots, s_n)$, $s_r = 0, 1, \ldots, 4$, $r = 1, \ldots, n$, converges to function $u(t,x) \in C^{0,4}_{t,x}\left(G_{[0,T]}\right)$. Under theorem about convergence of method of weak approximation [2] the function $u(t,x)$ is a solution of the problem (17), (3) and $u(t,x) \in C^{1,4}_{t,x}\left(G_{[0,T]}\right)$. Here

$$C^{p,q}_{t,x}\left(G_{[0,T]}\right) = \left\{u(t,x) \;\middle|\; \frac{\partial^m u}{\partial t^m}, \frac{\partial^{s_1+\cdots+s_n} u}{\partial x_1^{s_1}\cdots\partial x_n^{s_n}} \in C\left(G_{[0,T]}\right), m = 0,1,\ldots,p, \right.$$
$$\left. s_r = 0,1,\ldots,q,\ r = 1,\ldots,n\right\}.$$

At the same time the following estimations are valid for $(t,x) \in G_{[0,T]}$:

$$\left|D^s u(t,x)\right| \leqslant C,\ s = (s_1,\ldots,s_n),\ s_r = 0,1,\ldots,4,\ r = 1,\ldots,n. \tag{31}$$

4. Existence of the solution for inverse problem

Now we will prove that the pair of functions $u(t,x), \lambda(t,x)$, where $\lambda(t,x)$ is defined by (14) and satisfies condition (4), is the solution of the inverse problem. Function $u(t,x)$ is the solution of the direct problem. Substituting $u(t,x)$ into (1) and (3) we obtain that identities are hold.

From conditions (8), (9), (11), (14), (31), equation (17) it follows that functions $u(t,x)$, $\lambda(t,x)$ belong to the class

$$Z(T) = \left\{u(t,x), \lambda(t,x) \;\middle|\; u \in C^{1,4}_{t,x}\left(G_{[0,T]}\right), \lambda(t,x) \in C^{0,2}_{t,x}\left(G_{[0,T]}\right)\right\},$$

and satisfy the following inequality

$$\sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,\ldots,4,\\ r=1,\ldots,n}} \left|D^s u(t,x)\right| + \sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,2,\\ r=1,\ldots,n}} \left|D^s \lambda(t,x)\right| \leqslant C. \tag{32}$$

Let us prove that solution $u(t,x)$, $\lambda(t,x)$ of the direct problem satisfies conditions (5). Consequently we substitute $x_k = \alpha_k,\ k = 1, 2, \ldots, n$, to (17):

$$u_t(t, x_1, a_2^2) - \frac{\partial}{\partial t}\varphi_1(t, x_1) = b_1(t) \cdot \left(u_{x_1x_1}(t, x_1, a_2^2) - \frac{\partial^2}{\partial x_1^2}\varphi_1(t, x_1)\right) +$$
$$+ c_1(t) \cdot \left(u_{x_1}(t, x_1, a_2^2) - \frac{\partial}{\partial x_1}\varphi_1(t, x_1)\right) + (n-1) \cdot \Phi_0(t, x_1, a_2^2) \times$$
$$\times \left(b_1(t)u_{x_1x_1}(t, a_0) - b_1(t)\frac{\partial^2}{\partial x_1^2}\varphi_1(t, \alpha_1) + c_1(t)u_{x_1}(t, a_0) - c_1(t)\frac{\partial}{\partial x_1}\varphi_1(t, \alpha_1)\right) +$$
$$+ (n-2) \cdot \Phi_0(t, x_1, a_2^2) \cdot \left(\sum_{k=2}^{n}\left[b_k(t)u_{x_kx_k}(t, a_0) - b_k(t)\frac{\partial^2}{\partial x_k^2}\varphi_k(t, \alpha_k)\right] +\right.$$
$$\left.+ \sum_{k=2}^{n}\left[c_k(t)u_{x_k}(t, a_0) - c_k(t)\frac{\partial}{\partial x_k}\varphi_k(t, \alpha_k)\right]\right),$$
$$\ldots$$
$$u_t(t, a_{n-1}^1, x_n) - \frac{\partial}{\partial t}\varphi_n(t, x_n) = b_n(t) \cdot \left(u_{x_nx_n}(t, a_{n-1}^1, x_n) - \frac{\partial^2}{\partial x_n^2}\varphi_n(t, x_n)\right) +$$
$$+ c_n(t) \cdot \left(u_{x_n}(t, a_{n-1}^1, x_n) - \frac{\partial}{\partial x_n}\varphi_n(t, x_n)\right) + (n-1) \cdot \Phi_0(t, a_{n-1}^1, x_n) \times$$
$$\times \left(b_n(t)u_{x_nx_n}(t, a_0) - b_n(t)\frac{\partial^2}{\partial x_n^2}\varphi_n(t, \alpha_n) + c_n(t)u_{x_n}(t, a_0) - c_n(t)\frac{\partial}{\partial x_n}\varphi_n(t, \alpha_n)\right) +$$
$$+ (n-2) \cdot \Phi_0(t, a_{n-1}^1, x_n) \cdot \left(\sum_{k=1}^{n-1}\left[b_k(t)u_{x_kx_k}(t, a_0) - b_k(t)\frac{\partial^2}{\partial x_k^2}\varphi_k(t, \alpha_k)\right] +\right.$$
$$\left.+ \sum_{k=1}^{n-1}\left[c_k(t)u_{x_k}(t, a_0) - c_k(t)\frac{\partial}{\partial x_k}\varphi_k(t, \alpha_k)\right]\right).$$

Let us introduce the notations:

$$\psi_k(t, x_k) = u(t, a_{k-1}^1, x_k, a_{k+1}^2) - \varphi_k(t, x_k),\ k = 1, 2, \ldots, n.$$

Then we have the following system of equations:

$$\frac{\partial}{\partial t}\psi_1(t, x_1) = b_1(t) \cdot \frac{\partial^2}{\partial x_1^2}\psi_1(t, x_1) + c_1(t) \cdot \frac{\partial}{\partial x_1}\psi_1(t, x_1) +$$
$$+ (n-1) \cdot \Phi_0(t, x_1, a_2^2) \cdot \left(b_1(t)\frac{\partial^2}{\partial x_1^2}\psi_1(t, \alpha_1) + c_1(t)\frac{\partial}{\partial x_1}\psi_1(t, \alpha_1)\right) +$$
$$+ (n-2) \cdot \Phi_0(t, x_1, a_2^2) \cdot \sum_{k=2}^{n}\left(b_k(t)\frac{\partial^2}{\partial x_k^2}\psi_k(t, \alpha_k) + c_k(t)\frac{\partial}{\partial x_k}\psi_k(t, \alpha_k)\right),$$
$$\ldots$$
$$\frac{\partial}{\partial t}\psi_n(t, x_n) = b_n(t) \cdot \frac{\partial^2}{\partial x_n^2}\psi_n(t, x_n) + c_n(t) \cdot \frac{\partial}{\partial x_n}\psi_n(t, x_n) +$$
$$+ (n-1) \cdot \Phi_0(t, a_{n-1}^1, x_n) \cdot \left(b_n(t)\frac{\partial^2}{\partial x_n^2}\psi_n(t, \alpha_n) + c_n(t)\frac{\partial}{\partial x_n}\psi_n(t, \alpha_n)\right) +$$

$$+(n-2)\cdot\Phi_0(t,a_{n-1}^1,x_n)\cdot\sum_{k=1}^{n-1}\left(b_k(t)\frac{\partial^2}{\partial x_k^2}\psi_k(t,\alpha_k)+c_k(t)\frac{\partial}{\partial x_k}\psi_k(t,\alpha_k)\right), \tag{33}$$

$$\psi_k(0,x_k)=0,\ k=1,2,\ldots,n. \tag{34}$$

Let $\Psi=(\psi_1,\psi_2,\ldots,\psi_n)$ be the vector of solutions of the problem (33),(34). Vector $\Psi_0 = (0,0,\ldots,0)$ is the solution of the problem (33),(34). Let us prove that the solution is unique.

We will prove this by contradiction. Suppose there are two different solutions of the problem (33), (34), which satisfy (11), (31): $B^1 = (\beta_1^1,\beta_2^1,\ldots,\beta_n^1)\in C^{1,4}\left(G_{[0,T]}\right)$ and $B^2=(\beta_1^2,\beta_2^2,\ldots,\beta_n^2)\in C^{1,4}\left(G_{[0,T]}\right)$.

The difference

$$B=B^1-B^2=(\beta_1=\beta_1^1-\beta_1^2,\beta_2=\beta_2^1-\beta_2^2,\ldots,\beta_n=\beta_n^1-\beta_n^2) \tag{35}$$

is the solution of the following problem:

$$\begin{aligned}
&\frac{\partial}{\partial t}\beta_1(t,x_1)=b_1(t)\cdot\frac{\partial^2}{\partial x_1^2}\beta_1(t,x_1)+c_1(t)\cdot\frac{\partial}{\partial x_1}\beta_1(t,x_1)+\\
&+(n-1)\cdot\Phi_0(t,x_1,a_2^2)\cdot\left(b_1(t)\frac{\partial^2}{\partial x_1^2}\beta_1(t,\alpha_1)+c_1(t)\frac{\partial}{\partial x_1}\beta_1(t,\alpha_1)\right)+\\
&+(n-2)\cdot\Phi_0(t,x_1,a_2^2)\cdot\sum_{k=2}^{n}\left(b_k(t)\frac{\partial^2}{\partial x_k^2}\beta_k(t,\alpha_k)+c_k(t)\frac{\partial}{\partial x_k}\beta_k(t,\alpha_k)\right),\\
&\ldots\\
&\frac{\partial}{\partial t}\beta_n(t,x_n)=b_n(t)\cdot\frac{\partial^2}{\partial x_n^2}\beta_n(t,x_n)+c_n(t)\cdot\frac{\partial}{\partial x_n}\beta_n(t,x_n)+\\
&+(n-1)\cdot\Phi_0(t,a_{n-1}^1,x_n)\cdot\left(b_n(t)\frac{\partial^2}{\partial x_n^2}\beta_n(t,\alpha_n)+c_n(t)\frac{\partial}{\partial x_n}\beta_n(t,\alpha_n)\right)+\\
&+(n-2)\cdot\Phi_0(t,a_{n-1}^1,x_n)\cdot\sum_{k=1}^{n-1}\left(b_k(t)\frac{\partial^2}{\partial x_k^2}\beta_k(t,\alpha_k)+c_k(t)\frac{\partial}{\partial x_k}\beta_k(t,\alpha_k)\right),\\
&\qquad\beta_k(0,x_k)=0,\ k=1,2,\ldots,n.
\end{aligned} \tag{36}$$

We introduce nonnegative, nondecreasing in $[0,T]$ functions like this:

$$p_k^l(t)=\sup_{(\xi,x)\in G_{[0,t]}}\left|\frac{\partial^l}{\partial x_k^l}\beta_k(\xi,x_k)\right|,\ l=0,1,2,\ k=1,2,\ldots,n. \tag{37}$$

Under the principle of maximum for equations of system (36) we obtain

$$\begin{aligned}
&\left|\beta_1(\xi,x_1)\right|\leqslant C\cdot\left(p_1^2(t)+p_1^1(t)\right)\cdot t+C\cdot\sum_{k=2}^{n}\left(p_k^2(t)+p_k^1(t)\right)\cdot t,\\
&\left|\beta_2(\xi,x_2)\right|\leqslant C\cdot\left(p_2^2(t)+p_2^1(t)\right)\cdot t+C\cdot\sum_{k=1,k\neq 2}^{n}\left(p_k^2(t)+p_k^1(t)\right)\cdot t,\\
&\ldots\\
&\left|\beta_n(\xi,x_n)\right|\leqslant C\cdot\left(p_n^2(t)+p_n^1(t)\right)\cdot t+C\cdot\sum_{k=1}^{n-1}\left(p_k^2(t)+p_k^1(t)\right)\cdot t,\\
&0\leqslant\xi\leqslant t,\ 0\leqslant t\leqslant T,\ x_k\in\mathbb{R}^1,k=1,2,\ldots,n.
\end{aligned} \tag{38}$$

We differentiate k-th equation from system (36) with respect to $x_k,\ k = 1, 2, \dots, n$, one and two times. Under the principle of maximum for the obtained equations we get the estimations:

$$
\begin{aligned}
&\left|\frac{\partial^l}{\partial x_1^l}\beta_1(\xi, x_1)\right| \leqslant C \cdot \left(p_1^{l+2}(t) + p_1^{l+1}(t)\right) \cdot t + C \cdot \sum_{k=2}^{n}\left(p_k^2(t) + p_k^1(t)\right) \cdot t,\\
&\dots\\
&\left|\frac{\partial^l}{\partial x_n^l}\beta_n(\xi, x_n)\right| \leqslant C \cdot \left(p_n^{l+2}(t) + p_n^{l+1}(t)\right) \cdot t + C \cdot \sum_{k=1}^{n-1}\left(p_k^2(t) + p_k^1(t)\right) \cdot t,\\
&0 \leqslant \xi \leqslant t,\ 0 \leqslant t \leqslant T,\ x_k \in \mathbb{R}^1, k = 1, 2, \dots, n.
\end{aligned}
\tag{39}
$$

We apply $\sup\limits_{(\xi,x)\in G_{[0,t]}}$ to both parts of inequalities (38), (39), sum up the obtained results. Next under the nonnegativity of functions (37) we have:

$$
\sum_{l=0}^{2}\sum_{k=1}^{n} p_k^l(t) \leqslant C \cdot t \cdot \sum_{l=0}^{2}\sum_{k=1}^{n} p_k^l(t),\ 0 \leqslant t \leqslant T.
$$

Hence it follows that the following equation is hold by $t \in [0, \theta]$, where $\theta < \dfrac{1}{C}$.

$$
\sum_{l=0}^{2}\sum_{k=1}^{n} p_k^l(t) = 0.
$$

Taking into consideration the properties of functions (37) we get

$$
\beta_k(t, x_k) = 0,\ k = 1, 2, \dots, n,\ x_k \in \mathbb{R}^1,\ 0 \leqslant t \leqslant \theta.
$$

Thinking the same way, when $t \in [\theta, 2\theta]$, we obtain

$$
\beta_k(t, x_k) = 0,\ k = 1, 2, \dots, n,\ x_k \in \mathbb{R}^1,\ \theta \leqslant t \leqslant 2\theta.
$$

After finite number of steps we have

$$
\beta_k(t, x_k) \equiv 0,\ k = 1, 2, \dots, n,\ x_k \in \mathbb{R}^1,\ 0 \leqslant t \leqslant T. \tag{40}
$$

From formulas (35) and (40) it follows that

$$
\beta_k^1 - \beta_k^2 \equiv 0,\quad k = 1, 2, \dots, n,\ x_k \in \mathbb{R}^1,\ 0 \leqslant t \leqslant T,
$$

that is

$$
\beta_k^1 = \beta_k^2,\ k = 1, 2, \dots, n,\ x_k \in \mathbb{R}^1,\ 0 \leqslant t \leqslant T.
$$

Thereby the supposed solutions are equal in all area $G_{[0,T]}$. That's why the solution of the problem (33), (34) is unique.

Because of the problem (33),(34) has only zero solution, then conditions (5) are satisfied. Therefore the pair of functions $u(t, x)$, $\lambda(t, x)$ is the solution of the main inverse problem (1)–(5).

Thus the following theorem has been proved.

Theorem 1. *Let conditions* (6)–(11) *be hold. Then solution* $u(t, x)$, $\lambda(t, x)$ *of inverse problem* (1)–(5) *exists and satisfies* (32) *in class* $Z(T)$.

5. Uniqueness of the inverse problem solution

Let conditions (8)–(11), (32) be fulfilled. We will prove uniqueness of solution of problem (1)–(5) by contradiction.

Suppose $\{u(t,x), \lambda(t,x)\}$ and $\{\tilde{u}(t,x), \tilde{\lambda}(t,x)\}$ are two classical solutions of problem (1)–(5). The pair of functions $u(t,x)$, $\lambda(t,x)$ is the solution which is defined by equation (14) and satisfies condition (4). And pair of functions $\tilde{u}(t,x)$, $\tilde{\lambda}(t,x) = \sum\limits_{k=1}^{n} \tilde{\lambda}_k(t,x_k)$ is some another solution of problem (1)–(5) which satisfies condition (32). Then the following formulas are hold.

$$u_t(t,x) = L\left(u(t,x)\right) + f(t,x)\cdot\lambda(t,x),$$

$$\tilde{u}_t(t,x) = L\left(\tilde{u}(t,x)\right) + f(t,x)\cdot\tilde{\lambda}(t,x),$$

$$u(0,x) = u_0(x),\ \tilde{u}(0,x) = u_0(x),$$

$$u(t, a^1_{k-1}, x_k, a^2_{k+1}) = \varphi_k(t,x_k),\ k = 1,2,\ldots,n,$$

$$\tilde{u}(t, a^1_{k-1}, x_k, a^2_{k+1}) = \varphi_k(t,x_k),\ k = 1,2,\ldots,n.$$

We introduce the notations:

$$w(t,x) = u(t,x) - \tilde{u}(t,x),\ \gamma(t,x) = \sum_{k=1}^{n}\gamma_k(t,x_k) = \lambda(t,x) - \tilde{\lambda}(t,x).$$

So the pair of functions $w(t,x), \gamma(t,x)$ is the solution of Cauchy problem:

$$w_t(t,x) = L\left(w(t,x)\right) + f(t,x)\cdot\gamma(t,x), \tag{41}$$

$$w(0,x) = 0,\ w(t, a^1_{k-1}, x_k, a^2_{k+1}) = 0,\ k = 1,2,\ldots,n. \tag{42}$$

Consequently we substitute $x = a_0,\ x_k = \alpha_k,\ k = 1,2,\ldots,n$, to equation (41). Using (42) we evaluate the coefficient at function $f(t,x)$:

$$\gamma(t,x) = \sum_{k=1}^{n}\gamma_k(t,x_k) = -\sum_{i=1}^{n}\frac{\sum\limits_{\substack{k=1,\\k\neq i}}^{n} b_k(t) w_{x_k x_k}(t, a^1_{i-1}, x_i, a^2_{i+1}) + \sum\limits_{\substack{k=1,\\k\neq i}}^{n} c_k(t) w_{x_k}(t, a^1_{i-1}, x_i, a^2_{i+1})}{f(t, a^1_{i-1}, x_i, a^2_{i+1})}.$$

Next we substitute the latter formula to (41) and obtain

$$w_t = L\left(w(t,x)\right) + \sum_{i=1}^{n}\Phi_i(t,x)\cdot\left(\sum_{\substack{k=1,\\k\neq i}}^{n} b_k(t) w_{x_k x_k}(t, a^1_{i-1}, x_i, a^2_{i+1}) + \right.$$
$$\left. + \sum_{\substack{k=1,\\k\neq i}}^{n} c_k(t) w_{x_k}(t, a^1_{i-1}, x_i, a^2_{i+1})\right), \tag{43}$$

$$w(0,x) = 0. \tag{44}$$

Let us introduce nonnegative, nondecreasing in $[0,T]$ functions:

$$V_s(t) = \sup_{(\xi,x)\in G_{[0,t]}}\left|D^s w(\xi,x)\right|,\ s = (s_1,\ldots,s_n),\ s_r = 0,1,2,\ r = 1,\ldots,n.$$

Taking (8), (32) into account, under the principle of maximum we get that for the equation (43) the following inequality is valid.

$$\left|w(\xi,x)\right| \leqslant C\cdot\xi\cdot \sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,2,\\ r=1,\ldots,n}} V_s(t), \quad 0<\xi\leqslant t,\ (t,x)\in G_{[0,T]}.$$

Under nonnegativity of functions $V_s(t)$ it follows that

$$V_0(t)\leqslant C\cdot t\cdot \sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,2,\\ r=1,\ldots,n}} V_s(t), \quad 0\leqslant t\leqslant T. \tag{45}$$

Applying differential operator D^s with $s=(s_1,\ldots,s_n)$, $s_r=0,1,2$, $r=1,\ldots,n$, to (43), (44) under the principle of maximum we obtain the estimations:

$$V_s(t)\leqslant C\cdot t\cdot \sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,2,\\ r=1,\ldots,n}} V_s(t), \quad 0\leqslant t\leqslant T. \tag{46}$$

Summing all estimations (45) and (46) we get

$$\sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,2,\\ r=1,\ldots,n}} V_s(t)\leqslant C\cdot t\cdot \sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,2,\\ r=1,\ldots,n}} V_s(t), \quad 0\leqslant t\leqslant T.$$

Hence the following estimation is valid by $t\in[0,\xi]$, where $\xi<\dfrac{1}{C}$.

$$\sum_{\substack{s=(s_1,\ldots,s_n),\\ s_r=0,1,2,\\ r=1,\ldots,n}} V_s(t)=0.$$

Therefore $w(t,x)=0$ by $(t,x)\in G_{[0,\xi]}$. Replicating the reasoning by $t\in[\xi,2\xi]$ we obtain that $w(t,x)=0$ by $(t,x)\in G_{[0,2\xi]}$. In finite number of steps we will prove that $w(t,x)\equiv 0$ on $G_{[0,T]}$. It means that $u(t,x)=\tilde{u}(t,x)$ on $G_{[0,T]}$.

From equation (41) and conditions (42) we see that

$$f(t,x)\cdot\gamma(t,x)=0. \tag{47}$$

Let us consider (47) when $x=a_0$, $x_k=\alpha_k,\ k=1,2,\ldots,n$:

$$\begin{aligned}
&f(t,a_0)\cdot\big(\gamma_1(t,\alpha_1)+\ldots+\gamma_n(t,\alpha_n)\big)=0,\\
&f(t,x_1,a_2^2)\cdot\big(\gamma_1(t,x_1)+\gamma_2(t,\alpha_2)+\ldots+\gamma_n(t,\alpha_n)\big)=0,\\
&\ldots\\
&f(t,a_{n-1}^1,x_n)\cdot\big(\gamma_1(t,\alpha_1)+\ldots+\gamma_{n-1}(t,\alpha_{n-1})+\gamma_n(t,x_n)\big)=0.
\end{aligned}$$

Under conditions (8) for source function the following formulas are hold.

$$\gamma_1(t,\alpha_1)+\gamma_2(t,\alpha_2)+\ldots+\gamma_n(t,\alpha_n)=0, \tag{48}$$

$$\begin{aligned} &\gamma_1(t, x_1) + \gamma_2(t, \alpha_2) + \ldots + \gamma_n(t, \alpha_n) = 0 \\ &\gamma_1(t, \alpha_1) + \gamma_2(t, x_2) + \ldots + \gamma_n(t, \alpha_n) = 0, \\ &\ldots \\ &\gamma_1(t, \alpha_1) + \ldots + \gamma_{n-1}(t, \alpha_{n-1}) + \gamma_n(t, x_n) = 0. \end{aligned} \tag{49}$$

Summing up equalities (49) we have

$$\sum_{k=1}^{n} \gamma_k(t, x_k) + (n-1) \sum_{k=1}^{n} \gamma_k(t, \alpha_k) = 0.$$

Hence under (48) it follows that $\gamma(t, x) = \sum\limits_{k=1}^{n} \gamma_k(t, x_k) = 0$. That is

$$\lambda(t, x) = \tilde{\lambda}(t, x),\ t \in [0, T].$$

Thereby we have proved the following theorem.

Theorem 2. *The solution* $u(t, x), \lambda(t, x)$ *satisfying* (32) *of the problem* (1)–(5) *is unique in class* $Z(T)$.

Theorems 1 and 2 are followed by the theorem 3.

Theorem 3. *Let conditions* (6)–(11) *be hold. Then in class* $Z(T)$ *the problem* (1)–(5) *has unique solution* $u(t, x), \lambda(t, x)$ *which satisfies inequality* (32).

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Об одной задаче идентификации функции источника специального вида для многомерного параболического уравнения с данными Коши

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В статье доказаны существование и единственность решения задачи идентификации функции источника специального вида для многомерного параболического уравнения с данными Коши.

Ключевые слова: обратная задача, идентификация функции источника, многомерное параболическое уравнение, метод слабой аппроксимации, существование и единственность решения, задача Коши.