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On Normal Closures of Involutions in the Group of Limited Permutations

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We study the group $G = Lim(N)$ of limited permutations of a set N of all natural numbers. Found the link between the dispersion subsets of a set N and normal subgroups of G.

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Introduction

Let N be the set of all natural numbers, Z be the set of all integers, M is any of these sets. The $S(M)$ will denote the group of all permutations of the set M.

Definition 1. *Permutation $g \in S(M)$ is called limited if*

$$w(g) = \max_{\alpha \in M} |\alpha - \alpha^g| < \infty.$$

If g, h are limited permutations, so the same is about the permutations g^{-1} and gh, as $w(g^{-1}) = w(g),\ w(gh) \leqslant w(g) + w(h)$. Thus set

$$Lim(M) = \{x \,|\, x \in S(M), w(x) < \infty\}$$

form a group, which is a natural extension of a locally finite group $Fin(M)$ of all finitary permutations of the set M, i.e. such permutation $y \in S(M)$, for which the set $\{\alpha \,|\, \alpha \in M, \alpha^y \neq \alpha\}$ is finite.

In the work of N. M. Suchkov [1] an example of the mixed group $H = AB$ was first built, where A, B is periodic (and even locally-finite) subgroups. Then in [2, 3] it was found that

$$H = \langle g \,|g \in Lim(Z), |g| < \infty\rangle,$$

any countable free group and Aleshin 2-group isomorphically embeddable into the group H and

$$Lim(Z) = H \leftthreetimes \langle d\rangle,$$

where d-shift, $\alpha^d = \alpha + 1$ for any $\alpha \in Z$.

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In their work [4] N. M. Suchkov and N. G. Suchkova proved the factorization of the whole group $Lim(N)$ by two locally-finite subgroups and it is shown that the group $Lim(M)$ is generated by the permutations $x \in S(M)$, for which $w(x) = 1$.

These generators are either involutions, in which decomposition into independent cycles only transpositions of the $(\alpha\,\alpha+1)$, $\alpha \in M$ or $M = Z$ and $x \in \{d, d^{-1}\}$. The relation between groups $Lim(N)$ and $Lim(Z)$ is found in [5]. Assuming that permutations of the group $S(N)$ have identical influence on the set $Z \setminus N$, we get a natural embedding of $S(N) < S(Z)$. Denote by t the involution of the groups $S(Z)$ for which $\alpha^t = -\alpha (\alpha \in Z)$. It is proved that

$$H = Fin(Z)(Lim(N) \times (Lim(N))^t).$$

From this congruence it follows that in the study of normal structure of the group $Lim(Z)$ is the defining description of normal subgroups of the group $Lim(N)$.

The first result in this direction is obtained in [5]. To formulate it is necessary to provide some definitions being introduced in this work. Let

$$L = \{\mu_1, \mu_2, \ldots, \mu_n, \ldots\}$$

is an infinite subset of N, where $\mu_1 < \mu_2 < \cdots < \mu_n < \ldots$; m is a fixed natural number. By definition, elements of μ_i and μ_j are equivalent, if $i = j$, or when $i < j\,(j < i)$ all inequations are fulfilled $\mu_{k+1} - \mu_k \leqslant m$; $i \leqslant k \leqslant j-1\,(j \leqslant k \leqslant i-1)$. It is easy to see that this relation is indeed an equivalence relation, therefore it induces a decomposition of the set L into equivalence classes. This partition is called m-partition. Let $B_m(L)$be the set of all equivalence classes of elements of the set L.

Definition 2. *The set L is called m-dispersion, if all classes of the set $B_m(L)$ are finite and are completely m-dispersion if*

$$c_m = \max_{A \in B_m(L)} |A| < \infty.$$

The set L is called (completely) dispersion, if it is (completely) m-dispersion for every natural m.

The example of completely dispersion set is the set L, for elements of which the following inequations are used

$$\mu_2 - \mu_1 < \mu_3 - \mu_2 < \cdots < \mu_n - \mu_{n-1} < \mu_{n+1} - \mu_n < \ldots .$$

Let $\mu_n + 1 < \mu_{n+1} (n = 1, 2, \ldots)$ and

$$a = (\mu_1\,\mu_1+1)(\mu_2\,\mu_2+1)\ldots(\mu_n\,\mu_n+1)\ldots$$

is the decomposition of an involution a into independent transpositions. The main result of [5] is the theorem according to which the normal closure of the involution of a in the group $Lim(N)$ if and only if locally finite, when L is an completely dispersion set.

Three hypotheses about normal subgroups of the group $Lim(N)$ are provided ibid.

In this article one of these hipotheses is proved; namely, the following theorem is the main result of the present paper.

Theorem 1. *An involution a if and only if contained in a proper normal subgroup of the group $Lim(N)$ when L is a dispersion set. If L is dispersion, but is not completely dispersion set, then $\langle a^g | g \in Lim(N)\rangle$ is a mixed group.*

All the designations used in this work are either discussed, or standard [6].

1. Preliminary results

Let γ, ε be integers and $\gamma \leqslant \varepsilon$. Let us call the set

$$U_{\gamma}^{\varepsilon} = \{\beta | \beta \in Z, \, \gamma \leqslant \beta \leqslant \varepsilon\}$$

a segment of integers; γ is the left end of the segment, ε is the right. In particular, $U_{\gamma}^{\gamma} = \{\gamma\}$.

For each $m \in N$, $\alpha \in L$ let be

$$V_{\alpha}^{m} = U_{\alpha-m}^{\alpha+m} \bigcap N, \; E_m = \bigcup_{\alpha \in L} V_{\alpha}^{m}.$$

Lemma 1. *If the set L is dispersion, then the set E_m is 1-dispersion for every natural m.*

Proof. If the Lemma is wrong, then such integers m and γ will be found that will prove that 1-decomposition of the E_m set contains an infinite class $U_{\gamma} = \{\beta \,|\, \beta \in N, \, \beta \geqslant \gamma\}$. Let $\mu_i > \gamma$, then the union $V_{\mu_i}^{m} \bigcup V_{\mu_{i+1}}^{m}$ includes a segment of integers with μ_i, μ_{i+1} endpoints. Therefore, $2m$ is a decomposition of the set L contains an infinite equivalence class with representative μ_i We have come to a contradiction with the dispersion of the set L. The Lemma is proved. □

For brevity let us define $G = Lim(N)$ and for each of dispersion set of L we define the subgroup $Q = Q(L)$. As to Lemma 1 each set E_m is split into segments

$$W_{m1}, \, W_{m2}, \ldots, \, W_{mn}, \ldots$$

of integers and if β_{mn} is the right end of the segment W_{mn}, and α_{mn+1} is the left end of the segment W_{mn+1}, then $\alpha_{mn+1} > \beta_{mn} + 1 \, (n = 1, 2, ...)$; each segment is W_{mn} is included into some interval $W_{m+1\,s}$. Let

$$Q_m = \{x \,|\, x \in G; \, W_{mn}^{x} = W_{mn} \, (n = 1, 2, \ldots); \, \beta^{x} = \beta(\beta \in N \setminus E_m)\}.$$

Obviously, Q_m is a subgroup of G and $Q_m \leqslant Q_{m+1}$, $m = 1, 2, \ldots$. Let, finally,

$$Q = Q(L) = \bigcup_{m \in N} Q_m$$

Lemma 2. *Q is proper normal subgroupin the group G.*

Proof. Let $1 \neq h \in Q$; $g \in G$ and $w(g) = k$. From the definition of the group Q, it follows that there are such natural m that the element h contains the subgroup Q_m. We claim that $h^g \in Q_t$, where $t = m + k + 1$. Indeed, consider the decomposition of permutation h into independent cycles. Since h leaves untouched the segments of $W_{mn} \, (n = 1, 2, \ldots)$ and acts identically on the integers which are not contained in these segments, then all the cycles are finite. If $x = (\gamma_1 \ldots \gamma_s)$ is one of these cycles is $(s > 1)$, then $\gamma_1, \ldots, \gamma_s$ are contained in some interval W_{mn}, which coincides with the union of several segments

$$V_{\mu_q}^{m}, \, V_{\mu_{q+1}}^{m}, \ldots, \, V_{\mu_e}^{m}.$$

We fix any number of γ_i of set $\{\gamma_1, \ldots, \gamma_s\}$. Then $\gamma_i \in V_{\mu_j}^{m}$ for some index j, $q \leqslant j \leqslant e$; and since $\gamma_i^{x} = \gamma_i^{g}$ and $|\gamma_i - \gamma_i^{g}| \leqslant w(g) = k$, then $\gamma_i^{g} \in V_{\mu_j}^{t}$. Next, segment W_{mn} is part of the segment

$$V_{\mu_q}^{t} \bigcup V_{\mu_{q+1}}^{t} \bigcup \ldots \bigcup V_{\mu_e}^{t}.$$

In turn, this segment is contained in some segment W_{td}, $d \in N$. Therefore, all elements of the cycle $x^g = (\gamma_1^g \dots \gamma_s^g)$ belong to W_{td}, where due to the definition of the group Q_t we deduce that $x^g \in Q_t$ and $h^g \in Q_t$. Thus, Q is a normal subgroup of G. It remains to show that Q is proper subgroup of G. In fact, in the course the corroboration we have seen that any permutation of the subgroup Q is decomposed into finite independent cycles.

Therefore an infinite cycle

$$y = (\dots 2n \dots 4\,2\,1\,3 \dots 2n-1 \dots)$$

is not contained in Q, but since $w(y) = 2$, then $y \in G$. Thus, $Q \neq G$. The Lemma is proved. □

Lemma 3. *If z is involution and f is a triple of the cycle of the alternating group A_4, $zz^f z^{f^2} = 1$.*

Proof. We have $A_4 = (\langle z\rangle \times \langle u\rangle) \leftthreetimes \langle f\rangle$, where $|u| = 2$. Thus f transitively permutes the involution z, u, zu whose product equals 1. Therefore, the Lemma is correct. □

In further calculations we will use the well-known and easily verifiable

Assertion 1. Let g, h are permutation of some set. If

$$g = \dots(\dots \alpha_1\, \alpha_2 \dots)\dots$$

is decomposition of g into independent cycles, then

$$g^h = h^{-1}gh = \dots(\dots \alpha_1^h\, \alpha_2^h \dots)\dots.$$

Lemma 4. *Let $y = (\varepsilon_1\, \varepsilon_2)(\varepsilon_3\, \varepsilon_4)(\varepsilon_5\, \varepsilon_6)$ is decomposition of the permutation y into independent transpositions, $f = (\varepsilon_3\, \varepsilon_4\, \varepsilon_5)$. Then $yy^f y^{f^2} = (\varepsilon_1\, \varepsilon_2)$.*

Proof. The elements $z = (\varepsilon_3\, \varepsilon_4)(\varepsilon_5\, \varepsilon_6)$, f generate a group isomorphic to alternating grope A_4, all elements of which commute with the transposition $(\varepsilon_1\, \varepsilon_2)$. Therefore, in view of Lemma 3 we have

$$y = (\varepsilon_1\, \varepsilon_2)z,\ y^f = (\varepsilon_1\, \varepsilon_2)z^f,\ y^{f^2} = (\varepsilon_1\, \varepsilon_2)z^{f^2},$$
$$yy^f y^{f^2} = (\varepsilon_1\, \varepsilon_2)^3 zz^f z^{f^2} = (\varepsilon_1\, \varepsilon_2).$$

The Lemma is proved. □

Lemma 5. *Let*

$$c = (\beta_1\beta_1+1)(\beta_2\beta_2+1)\dots(\beta_n\beta_n+1)\dots$$

is the decomposition of permutation c of the group G into independent transpositions If all of the inequalities are fulfilled

$$6 \leqslant \beta_{n+1} - \beta_n \leqslant m\, (n = 1, 2, \dots),$$

where m is some fixed natural number, then the normal closure of $B(c) = \langle c^g | g \in G\rangle$ the involution of c in the group G contains a group $Fin(N)$ of all finitary permutations of the set N.

Proof. Since the group $Fin(N)$ coincides with the normal closure of any of its transposition, in order to prove the Lemma it is enough to show that $B(c)$ contains a transposition $(\beta_1\, \beta_1+1)$ from the decomposition of permutation c. In fact, since $\beta_{n+1} - \beta_n \geqslant 6$ for all natural n, the transpositions of the permutation's decomposition

$$l = (\beta_1\, \beta_1+2)(\beta_1+1\, \beta_1+3)(\beta_2\, \beta_2+2)(\beta_2+1\, \beta_2+3)\dots(\beta_n\, \beta_n+2)(\beta_n+1\, \beta_n+3)\dots$$

is independent, and in the process $w(l) = 2$, in particular $l \in G$. Therefore, the group $B(c)$ contains an involution c^e. Using the Assertion 1 we get

$$c_1 = c^e = (\beta_1 + 2\,\beta_1 + 3)(\beta_2 + 2\,\beta_2 + 3)\dots(\beta_n + 2\,\beta_n + 3)\dots .$$

Now let

$$s = (\beta_1 + 2\,\beta_2)(\beta_1 + 3\,\beta_2 + 1)(\beta_2 + 2\,\beta_3)(\beta_2 + 3\,\beta_3 + 1)\dots(\beta_n + 2\,\beta_{n+1})(\beta_n + 3\,\beta_{n+1} + 1)\dots .$$

By condition of the Lemma $\beta_{n+1} - \beta_n \leqslant m\,(n = 1, 2, \dots)$, therefore, $w(s) < m$. Thus, $c_1^s \in B(c)$ and we get

$$c_1^s = (\beta_2\,\beta_2 + 1)(\beta_3\,\beta_3 + 1)\dots(\beta_{n+1}\,\beta_{n+1} + 1)\dots .$$

Thus, $cc_1^s = (\beta_1\,\beta_1 + 1)$ is a contained in $B(c)$. The Lemma is proved. □

Assertion 2([5], Lemma 7). Let $\{\alpha_1, \dots, \alpha_k\}$ is a subset of set N and $\alpha_i + 1 < \alpha_{i+1}$, $1 \leqslant i \leqslant k - 1$. If

$$b = (\alpha_1\,\alpha_1 + 1)(\alpha_2\,\alpha_2 + 1)\dots(\alpha_{k-1}\,\alpha_{k-1} + 1)(\alpha_k\,\alpha_k + 1)$$

is the decomposition involutive permutation $b \in Fin(N)$ into independent of transpositions,

$$u = (\alpha_1\,\alpha_2\,\dots\,\alpha_{k-1}\,\alpha_k\,\alpha_k + 1\,\alpha_{k-1} + 1\,\dots\,\alpha_2 + 1\,\alpha_1 + 1)$$

is cycle, then the permutation bb^u has the order k.

2. Proof of the Theorem 1

Let us proceed to the direct corroboration of the theorem formulated at the end of the introduction. Initially, it will prove the first part.

Suppose that L is a dispersion set. Then from the constrution in Section 1 of the group

$$Q = Q(L) = \bigcup_n Q_n$$

and Lemma 2 it follows that the involution a belong to the subgroup of Q_1, which is contained in proper normal subgroup Q of G.

Conversely, suppose that a is contained in proper normal in $G = Lim(N)$ subgroup. Obviously, is equivalent to, subgroup $B(a) = \langle a^g \mid g \in G\rangle$ is a proper subgroup of G. Suppose that the set L is not dispersion. This means that there is such a natural integer m_0 that the set $B_{m_0}(L)$ contains an infinite class of A. Then if μ_γ is the minimal number of the set A, then from definition it follows that $\mu_{i+1} - \mu_i \leqslant m_0$ for all $i \geqslant \gamma$. Hence we deduce that $B_m(L)$ consists of a single class $\{L\}$, if $m > \max(m_0, \mu_\gamma)$. Fix $m_1 > m$. Thus,

$$\mu_{n+1} - \mu_n \leqslant m_1\,(n = 1, 2, \dots)\,.$$

Now let us prove that $B(a) = G$. Then we get a contradiction to our assumption $B(a) \neq G$ and the first part of the theorem will be proved. Firstly it should be noted, that in the group $B(a)$ we can find a permutation

$$c = (\beta_1\,\beta_1 + 1)(\beta_2\,\beta_2 + 1)\dots(\beta_n\,\beta_n + 1)\dots ,$$

that for some natural m all the inequations are fulfilled

$$6 \leqslant \beta_{n+1} - \beta_n \leqslant m \,(n = 1, 2, \dots). \tag{1}$$

Indeed, we will split the transpositions from decomposition a into triples:

$$a = (\mu_1\, \mu_1 + 1)(\mu_2\, \mu_2 + 1)(\mu_3\, \mu_3 + 1), \dots (\mu_{3k+1}\, \mu_{3k+1} + 1)(\mu_{3k+2}\, \mu_{3k+2} + 1)\,(\mu_{3k+3}\, \mu_{3k+3} + 1) \dots .$$

Let

$$t = (\mu_2\, \mu_2 + 1\, \mu_3) \dots (\mu_{3k+2}\, \mu_{3k+2} + 1\, \mu_{3k+3}) \dots .$$

Since $\mu_{n+1} - \mu_n \leqslant m_1$, therefore $w(t) \leqslant m_1$, and it means that, $t \in G$. Therefore, if $c = a\, a^t\, a^{t^2}$, then $c \in B(a)$ and by Lemma 4

$$c = (\mu_1\, \mu_1 + 1) \dots (\mu_{3k+1}\, \mu_{3k+1} + 1) \dots .$$

Thus from the inequation $2 \leqslant \mu_{n+1} - \mu_n \leqslant m_1$ it easily implies that $6 \leqslant \mu_{3k+4} - \mu_{3k+1} \leqslant 3m_1 = m$. Assuming $\beta_1 = \mu_1,\ \beta_2 = \mu_4,\ \dots,\ \beta_n = \mu_{3n-2}, \dots$, we get that the permutation c is the sought fore.

Let us note that the inclusion $c \in B(a)$ immediately implies that $B(c) \leqslant B(a)$, and therefore for the proof of the part 1 of the Theorem it is enough to establish the congruence $B(c) = G$. It was noted in the introduction that the group G is generated by involutions, in decomposition into independent transpositions of which only transpositions of the $(\alpha\, \alpha + 1)$ form take part. Since $Fin(N) < B(c)$ by Lemma 5, to prove the congruence $B(c) = G$ it is enough to show that if

$$x = (\gamma_1\, \gamma_1 + 1) \dots (\gamma_n\, \gamma_n + 1) \dots,$$

where $\gamma_{n+1} > \gamma_n + 1$ $(n = 1, 2, \dots)$, then $x \in B(c)$. Since $B(c)$ contains any finitary permutation, $x_n = (\gamma_1\, \gamma_1 + 1) \dots (\gamma_n\, \gamma_n + 1)$, then without loss of generality we can assume that $\gamma_1 > \beta_1$.

Denote $L_x = \{\gamma_n | \, n \in N\}$ and consider the case when the inequations are fulfilled for the elements of this set

$$\gamma_{n+1} - \gamma_n > 5m \ (n = 1, 2, \dots). \tag{2}$$

Let us split the set of $N \setminus \{1, 2, \dots, \beta_1\}$ into the segments of the integers is

$$\Delta_1 = U_{\beta_1+1}^{\beta_3},\ \dots,\ \Delta_n = U_{\beta_2 n+1}^{\beta_{2n+1}},\ \Delta_{n+1} = U_{\beta_{2n+1}+1}^{\beta_{2n+3}},\ \dots .$$

In virtue of the inequation (1)

$$|\Delta_n| = \beta_{2n+1} - \beta_{2n-1} = (\beta_{2n+1} - \beta_{2n}) + (\beta_{2n} - \beta_{2n-1)} \leqslant 2m\,.$$

From inequations (2) and $\gamma_1 > \beta_1$ this implies that

$$L_x \subset \bigcup_{n \in N} \Delta_n;$$

the intersection of $\Delta_n \cap L_x$ for every n is either empty or contains max one element; γ_i, γ_j is not contained in the adjacent segments for every $i \neq j$. Thus, there is such a sequence $j_1, j_2, \dots, j_n, \dots$, that $j_{n+1} - j_n > 1\,(n = 1, 2, \dots)$ and

$$\gamma_1 \in \Delta_{j_1},\, \gamma_2 \in \Delta_{j_2},\, \dots,\, \gamma_n \in \Delta_{j_n},\, \dots .$$

Let us define the permutation $\psi \in S(N)$ as follows: for $n = 1, 2, \dots$ assume

$$\gamma_n^{\psi} = \beta_{2j_n}, \quad \beta_{2j_n}^{\psi} = \gamma_n, \quad (\gamma_n + 1)^{\psi} = \beta_{2j_n} + 1, \quad (\beta_{2j_n} + 1)^{\psi} = \gamma_n + 1;$$

$$\gamma^{\psi} = \gamma, \text{ if } \gamma \notin \bigcup_{n \in N} \left(\{\gamma_n, \gamma_n + 1\} \cup \{\beta_{2j_n}, \beta_{2j_n} + 1\} \right) .$$

Since the elements γ_n, β_{2j_n} belong to the segment Δ_n and $|\Delta_n| \leqslant 2m$, then $w(\psi) < 2m$, i.e. $\psi \in G$. As to Assertion 1 we have

$$x^{\psi} = (\beta_{2j_1}\, \beta_{2j_1} + 1) \dots (\beta_{2j_n}\, \beta_{2j_n} + 1) \dots .$$

Now let

$$\alpha_1,\, \alpha_2,\, \dots,\, \alpha_n,\, \dots$$

be elements of the set $\{\beta_1, \dots, \beta_n, \dots\} \setminus \{\beta_{2j_1}, \dots, \beta_{2j_n}, \dots\}$, arranged in ascending order. From the above it follows that if $\alpha_i = \beta_k$, then α_{i+1} is element of the set $\{\beta_{k+1},\, \beta_{k+2}\}$, and therefore $\alpha_{i+1} - \alpha_i \leqslant \beta_{k+2} - \beta_k \leqslant 2m$. Here i is any natural number, $k = k(i)$. It's easy to deduce that the permutation

$$f = (\alpha_1\, \alpha_2\, \alpha_2 + 1) \dots (\alpha_{2n-1}\, \alpha_{2n}\, \alpha_{2n} + 1) \dots$$

is an element of the group G. Applying Lemmas 3, 4 we get the congruence $cc^f c^{f^2} = x^{\psi}$ from which it immediately follows that $x \in B(c)$.

Let us finally prove, that this inclusion is done in general case (without additional assumptions that for the elements of a set L_x inequations are fulfilled (2)). To do this, we fix any natural number $s > 5m$, and represent the permutation x as compositions of

$$x = x_1\, x_2\, \dots\, x_s\,,$$

where

$$x_i = (\gamma_i\, \gamma_i + 1)(\gamma_{i+s}\, \gamma_{i+s} + 1) \dots (\gamma_{i+ks}\, \gamma_{i+ks} + 1) \dots ,$$

$1 \leqslant i \leqslant s$. From the definition of the permutation of x it implies that if $L_{x_i} = \{\gamma_{i+ks} | k = 1, 2, \dots\}$, then the adjacent elements of this set an inequation is fulfilled

$$\gamma_{i+(k+1)s} - \gamma_{i+ks} > s > 5m\,,$$

which coincides with the inequation (2) for the neighbouring elements of the set L_x. But then by proved above, $x_i \in B(c)$, $1 \leqslant i \leqslant s$, and therefore $x \in B(c)$. The first part of the theorem is proved.

Let us prove the second part. Let L be a dispersion, but not comletely dispersion set. We need to show that the normal closure of $B(a)$ of an involution a of a group G contains an element of infinite order. Indeed, in view of the definition for some natural number r there are pairwise disjoint sets

$$L_n = \{\mu_{\alpha_n},\, \mu_{\alpha_n+1},\, \dots\, \mu_{\beta_n}\},$$

$n = 1, 2, \dots$ of L that $|L_n| > n$ and $\mu_{i+1} - \mu_i \leqslant r$ $(\alpha_n \leqslant i \leqslant \beta_n - 1)$. Let us define the permutation of the u set N by its decomposition into independent cycles u_n $(n = 1, 2, \dots)$. Let

$$u_n = (\mu_{\alpha_n}\, \mu_{\alpha_n+1}\, \dots\, \mu_{\beta_n}\, \mu_{\beta_n} + 1\, \mu_{\beta_n} + 1\, \mu_{\beta_n-1} + 1\, \dots \mu_{\alpha_n+1} + 1\, \mu_{\alpha_n} + 1)\,.$$

Then $w(u) \leqslant r$, i.e. $u \in G$. According to Assertion 2 the element $aa^u \in B(a)$ is decomposed into independent cycles which lengths is unbounded, and therefore $|aa^u| = \infty$. The theorem is proved.

In conclusion, let us put an example of be a dispersion, but not comletely dispersion set. Let

$$L_1 = \{2,3\},\ L_2 = \{4,5,6\},\ L_3 = \{8,9,10,11\},\ \dots,\ L_n = \{2^n, 2^n+1, \dots, 2^n+n\};$$

$$L = L_1 \bigcup L_2 \bigcup L_3 \bigcup \cdots \bigcup L_n \bigcup \dots .$$

If 2^n is the representative of the class $A = A(n,m) \in B_m(L)$, then from definition it follows that A contains a set L_n of the $(n+1)$-th element, and if $m < 2^n - (2^{n-1} + n - 1)$, then $A = L_n$. Hence we conclude that the set L is a dispersion, but not comletely dispersion set.

References

[1] N.M.Suchkov, An example of a mixed group factorized by two periodic subgroups, *Algebra i Logika*, **23**(1984), no. 5, 385–387 (in Russian).

[2] N.M.Suchkov, On subgroups of the product of locally finite groups, *Algebra i logika*, **24**(1985), no. 4, 408–413 (in Russian).

[3] N.M.Suchkov, On the group of limited permutations, Konstruktsii v algebre i logike, Tver', 1990, 84–89 (in Russian).

[4] N.M.Suchkov, N.G.Suchkova, On groups of limited permutations, *Journal of Siberian Federal University. Mathematics and physics*, **3**(2010), no. 2, 262–266 (in Russian).

[5] N.M.Suchkov, N.G.Suchkova, Normal subgroups of limited permuta tion groups, *Siberskie electronnnye matematich. izvestiya*, **12**(2015), 344–353 (in Russian).

[6] M.I.Kargapolov, Y.I.Merzlyakov, Fundamentals of the theory of groups, New York, Springer-Verlag, 1979.

О нормальных замыканиях инволюций в группе ограниченных подстановок

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Изучается группа $G = Lim(N)$ ограниченных подстановок множества N всех натуральных чисел. Найдена связь между рассеянными подмножествами множества N и собственными нормальными подгруппами группы G.

Ключевые слова: группа, ограниченные перестановки, рассеивание, нормальная подгруппа, инволюции.