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Multi-Agents' Temporal Logic using Operations of Static Agents' Knowledge

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Abstract. We study an agents' temporal logic with non-standard none-transitive temporal accessibility relations and operations of static agents' knowledge. The main mathematical problem we work with is existence of algorithms for solving satisfiability problem. The problem is resolved and the algorithm is found. Some open problems are suggested in the concluding part.

Keywords: temporal logic, multi-agency, non-classical logics, information, knowledge representation, deciding algorithms, decidability, computability.

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Introduction

We start from a short historical comments. Temporal Logic has been broadly used to cover all approaches to reasoning about time and temporal information. Formally, as a mathematical subject, it may be seen as a special modal logic (formalized in 1960 by Arthur Prior under the name Tense Logic). Important feature of temporal logic is working out instruments for study of information and its stable in time part, which usually is referred as knowledge. In terms of symbolic logic, the concept of knowledge may be dated to the end of 1950. At 1962 Hintikka wrote the book: *Knowledge and Belief*, the first book-length work to suggest using modalities to capture the semantics of knowledge. This book laid much of the groundwork for the subject, but a great deal of research has taken place since that time. One of logics in that line of research was temporal logic (cf. for historical outlook for reasonably close days Gabbay, Hodkinson, Reynolds [7,8], Goldblat [9], Goranko [10], van Benthem [32], Yde Venema [35]).

There were many diverse variations of temporal logic in the study. For example, since invention the linear temporal logic $\mathcal{LTL}$ with operation $\mathbf{U}$ – until – by Amir Pnueli that system was popular for applications and due to interesting mathematical base. Automaton technique to solve satisfiability in this logic was developed by Vardi [33,34]). From reasonably modern results concerning this logic I would mention the solution for admissibility problem for $\mathcal{LTL}$ in Rybakov [18,19], the basis for admissible rules of $\mathcal{LTL}$ was obtained in Babenyshev and Rybakov [3].

The unification problem for $\mathcal{LTL}$ was solved in [23]. Concerning applications of logical methods in AI and CS, the tools around temporal logic work well for analysis in multi-agent environment (cf. eg. [20,21]). Representation of knowledge via multi-agent environment is a popular area in Logic in Computer Science.

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That concerns diverse subjects of multi-agent environment – interaction and autonomy, effects of cooperation etc. For example tools for representation agents' interaction for the logic $\mathcal{LTL}$ of linear time were developed in Rybakov [20, 21]. In current time this logic was investigated from many viewpoints, in particular extensions of $\mathcal{LTL}$ for the case of non-transitive models, were studied in Rybakov [24, 25, 29] for the case of the interval versions of the logic. Also modelling multi-agent reasoning via temporal models was applied in Rybakov [22,26,28] for various versions of linear logic.

In this our paper we study a temporal agents' logic based on agents' non-transitive time with possible time overlaps of time intervals and gaps (lacunas) on temporal accessibility relations — so by nature intransitive one. New feature we consider here is also application of operations of static agents' knowledge — $K_j p$ with the meaning the agent j knows the fact p in the (only) current time state (which may mean that later j may lose the knowledge about p); that may generate interesting logical effects). We solve the satisfiability problem for this logic via new approach which successfully packed all limitations in previous technique in one new framework. The paper contains all necessary definitions and does not require external reading and study. In concluding part we describe some interesting open problems.

1. None-transitive temporal models

At first we just recall logical language for our research. As usual logical language (which we will use) consists of potentially infinite set of propositional letters P, Boolean logical operations, operation $\mathcal{N}$ (next), operations $\mathbf{U}_j$ (until) for some finite fixed amount of indexes $j \in J$, each j for agent j.

Besides in this paper we will use operations K_j, for agents j, with meaning the agent j knows the fact (information) which is encoded by a letter $p \in P$, so $K_j p$ be a formula. Admit that we have exactly k agents — $[1, \dots, k]$. The formation rules for compound formulas are as always: any letter from P is a formula; the set of all formulas is closed w.r.t. applications of Boolean logical operations, the unary operation $\mathcal{N}$ (next) and the binary operations $\mathbf{U}_j$ (until); $\varphi \mathbf{U}_j \psi$ to be read φ holds within the agent j accessibility relation until ψ will be true, $\mathcal{N}\varphi$ says φ is true in next temporal state.

But for operations K_j, the only expressions $K_j p$ for $p \in P$ are formulas; so we cannot iterate the operation K_j in formulas, to apply K_j to compound formulas (in order to avoid possible contradictory logical circles).

Now we formally introduce models with non-transitive time and with various time relations for distinct agents'. Inside non-classical logic and logic in computer science, for a while, the question if the time might to be transitive or not was not in a focus of considerations, and was so to say suspended. Since a time ago, we got to be interested to clarify this point and to consider mathematical approaches to model distinct features of view on if the time might be transitive or none-transitive. A set of possible motivations, which previously used, may be seen in Rybakov [24–26, 28, 29]. We define now non-transitive temporal frames – the main semantic object of our paper.

Definition 1. *A* temporal non-transitive frame

$$\mathcal{F} := \langle N, \{R_x^j \mid x \in N, j \in J\}, Nxt\rangle$$

is a tuple such that for all $x \in N$, R_x^j is the linear order on the interval $[x, a_x^j]$ for some $a_x^j \geqslant x$, $a_x^j \in N$, it might be also that R_x^j is the linear order on whole interval $[x, \infty)$. $\forall x, y \in N$, $x\ Nxt\ y \Leftrightarrow y = x + 1$.

To illustrate this definition, consider that it may happen that $xR_x^1 a_x$, $y \in (x, a_x)$ and $not(yR_y^1 a_x)$. So to say the state y is a state situated earlier than the state x but y remember even less as the state x remember. It is immediate to see that the union of the all relations

R^j_x makes a non-transitive relation: $xR^1_x a_x$ (cf. the meaning of a_x in the definition above) and, it may happen, for some y, $x < y < a_x$, so $(aR^1_x y)$, $(yR^1_y a_y)$ but $not(xR^1_y a_y)$. Useful instruments for modelling reasoning about current processes might be built via presence of different agents' accessibility relations which might be distinct for the same time state but different agents.

Consider the case when, it may happen that $R^{a_1}_x = [x, x+15]$, $R^{a_2}_x = [x, x+25]$, and again to emphasize the effects of non-transitivity it may happen $z \in [x, x+25]$, $R^{a_2}_z = [z, x+21]$, so $\neg(zR^{a_2}_z(x+25))$, but, e.g. for the state $(z+2)$, $(z+2)R^{a_2}_x x+25$.

Interesting new feature in our paper is that for each agent the operations of accessibly by time to be referred to each time point. Our time intervals (which agents remember) are distinct for distinct agents, arbitrary by length for each agent and are referred to each time point. In any time point s any agent j remember the only states situated inside an time interval $[s, a^j_s]$.

Definition 2. *For any frame $\mathcal{F}$, a model $\mathcal{M}$ on $\mathcal{F}$ is defined by introduction a valuation V on $\mathcal{F}$ for a set of propositional letters p: $V(p) \subseteq N$. Else for any $a \in N$, $p \in P$, $K(a,p)$ is a fixed subset of the set of all agents j which (the model consider) know p on a.*

The valuation V to be extended to all formulas as follows:

For any $a \in N$:

$(N,a) \Vdash_V p \quad \Leftrightarrow \quad p \in V(p)$

$(N,a) \Vdash_V \neg\varphi \quad \Leftrightarrow \quad (N,a) \nVdash_V \varphi.$

Definition 3. *For compound formulas*

$$(N,a) \Vdash_V (\varphi \ \wedge \ \psi) \quad \Leftrightarrow \quad ((N,a) \Vdash_V \varphi) \wedge ((N,a) \Vdash_V \psi);$$

$$(N,a) \Vdash_V (\varphi \ \vee \ \psi) \quad \Leftrightarrow \quad ((N,a) \Vdash_V \varphi) \vee ((N,a) \Vdash_V \psi);$$

$$(N,a) \Vdash_V (\varphi \ \rightarrow \ \psi) \quad \Leftrightarrow \quad ((N,a) \Vdash_V \psi) \vee ((N,a) \nVdash_V \psi);$$

for formulas of sort $\varphi\mathbf{U}_j\psi$ we define the truth values as follows:

$$(N,c) \Vdash_V (\varphi \ \mathbf{U}_j \ \psi) \quad \Leftrightarrow$$

$$\exists b \in N[(cR^j_c b) \wedge ((N,b) \Vdash_V \psi) \wedge \quad \forall y[(y \geqslant c \& y < b) \Rightarrow (N,y) \Vdash_V \varphi]];$$

$$(N,a) \Vdash_V \mathcal{N}\varphi \quad \Leftrightarrow \quad (N,a+1) \Vdash_V \varphi];$$

$$(N,a) \Vdash_V K_j p \quad \Leftrightarrow \quad j \in K(a,p).$$

Just in case, recall that for any formula φ, $(N,a) \Vdash_V \varphi$ denotes that *the formula φ is true (valid)* at the state a w.r.t. the valuation V. We see that the truth of any formula with main temporal operations $\mathbf{U}_j$ at a state a refers only to the unique accessibility relation R^j_a for a. Sometimes we will use also the notation $Nxt(a) = b$ or $Next(a) = b$ to say that $a \ Nxt \ b$.

Definition 4. *Our multi-agent temporal logic TL^{MA}_{NT} is the set of all formulas which are valid at any state of any model based at any temporal frame $\mathcal{F}$.*

The modal operations may, as usual, be defined via temporal ones. The modal operations $\Box_i$ (necessary for agent i) and $\Diamond_i$ (possible for agent i) might be defined via temporal operations as follows: $\Diamond_i p := \top U_i p$, $\Box_i p := \neg\Diamond_i\neg p$. It might be easily verified that then

$$(\mathcal{M},a) \Vdash_V \Diamond_i\varphi \Leftrightarrow \exists b \in \mathcal{N}[(aR^i_a b) \wedge (\mathcal{M},b) \Vdash_V \varphi];$$

$$(\mathcal{M},a) \Vdash_V \Box_i\varphi \Leftrightarrow \forall b \in \mathcal{N}[(aR^i_a b) \Rightarrow (\mathcal{M},b) \Vdash_V \varphi].$$

Below we give simple illustrating examples on possible expressiveness of the formulas in chosen language.

(1) The formula $\Diamond_1 p \wedge \neg\Diamond_2 p$ being true w.r.t. a valuation V says that the accessibility relation for the agent 2 has a hole, lacuna, which nonetheless is accessible for the agent 1.

(2) Total opposition for in all first interval of time:

$\varphi_{op} := [\Box_1 p \to \Box_2 \neg p] \wedge [\Box_2 p \to \Box_1 \neg p]$. This formula being evaluated via a valuation V says that these both agents are totally opposite in their opinion for stable facts at all states accessible for them.

(3) Recall: $\Diamond_1 p \wedge \Box_1(p \to \Diamond_1[\neg p \wedge \neg\Box_1\neg p]) \wedge \Diamond_1\Diamond_1\Box_1 p$. This formula says that the agent 1 always swapping its opinion about truth of p from true to false and vise versa, but then — somewhere at next time interval its decides p is always true.

Now we would like to show how we may model via chosen language various properties related to multi-agent reasoning.

Definition 5. *Consider the following formulas*

$$Plausible\ \varphi := \bigwedge_{i\in[1,k]} \Box_i[\bigwedge_{j\in[1,k]} \Diamond_j \varphi];$$

$$Safe\ \varphi := \bigwedge_{i\in[1,k]} \Box_i[\bigvee_{X, X\subseteq[1,k], ||X||>k/2, j\in X} [\Box_j \Diamond_j \varphi]];$$

$$Reliable\ \varphi := \bigwedge_{i\in[1,k]} \Box_i[\bigwedge_{j\in[1,k]} \Box_j \varphi].$$

Here notation coincides with intended meaning. For example, *Plausible* φ means that for any agent j for any state s accessible by time relation for j exists at least one agent for which a state s_1 is accessible where φ is true at s_1. So, φ is plausible – for any state a network for any agent j in j-future there is at least one agent which sees in its future a state where φ is true (so φ is plausible).

Also, note that the presence the operations K_j is important also. For example, it may happen that $(\mathcal{M}, a) \Vdash p$ and $(\mathcal{M}, a) \nVdash K_j p$ for some j, that means that p is true (objectively true) at a but j does not know it.

2. Preparation a technique, reduced forms

Here, in this section, we unavoidably recall material from our other papers to make current paper easy readable and to avoid to enforce the reader to look necessary information in other publications. Our aim is to show that the satisfiability problem for introduced logic is decidable. Usual technique based at filtration, usage temporal degree of formulas and dropping points do not work for this semantics since the relations maybe total non-transitive and rules for computation truth values of formulas with $\mathbf{U}_j$ are different from standard. We will use a modification of our old technique for reduction of formulas to rules (which we have already used earlier many times for different purposes (cf. e. g. [19, 21]) and transformation the latter ones to so-called reduced forms. We briefly recall this technique. A *rule* is an expression $\mathbf{r} := \varphi_1(x_1, \dots, x_n), \dots, \varphi_s(x_1, \dots, x_n) \ / \ \psi(x_1, \dots, x_n)$, where all $\varphi_k(x_1, \dots, x_n)$ and $\psi(x_1, \dots, x_n)$ are formulas constructed out of letters (variables) $x_1, \dots, x_n$.

Formulas $\varphi_k(x_1, \dots, x_n)$ are called *premises* and $\psi(x_1, \dots, x_n)$ is the *conclusion*. The rule $\mathbf{r}$ means that $\psi(x_1, \dots, x_n)$ (conclusion) follows (logically follows) from the assumptions $\varphi_1(x_1, \dots, x_n)$, ..., $\varphi_s(x_1, \dots, x_n)$. The definition of the validness of a rule is the same for any relational model. To recall it, assume that a model $\mathcal{M}$ and a rule $\mathbf{r}$ are given.

The rule $\mathbf{r} := \varphi_1(x_1,\ldots,x_n),\ldots,\ \varphi_s(x_1,\ldots,x_n)\ /\ \psi(x_1,\ldots,x_n)$, *is valid on the model* $\mathcal{M}$ *based at a frame* $\mathcal{F}$ *iff*

$$[\forall a\left((\mathcal{F},a)\Vdash_V \bigwedge_{1\leqslant i\leqslant s}\varphi_i\right)] \quad \Rightarrow \quad [\forall a\,((\mathcal{F},a)\Vdash_V \psi)]\,.$$

If $\forall a\left((\mathcal{F},a)\Vdash_V \bigwedge_{1\leqslant i\leqslant s}\varphi_i\right)$ *but* $\exists a\,((\mathcal{F},a)\nVdash_V \psi)$, *then we say that* $\mathbf{r}$ *is refuted in* $\mathcal{F}$ *by* V *and we denote this fact as* $\mathcal{F}\nVdash_V \mathbf{r}$.

Definition 6. *A rule* $\mathbf{r}$ *is valid (or true) on a frame* $\mathcal{F}$ *iff* $\mathbf{r}$ *is true (valid) on any model based on* $\mathcal{F}$.

Definition 7. *A formula* φ *is satisfiable iff the rule* $T/\neg\varphi$ *may be refuted in some model* $\mathcal{M}$.

The proof is obvious — immediately follows from definitions. Thus we have

Lemma 8. *If there is an algorithm verifying for any given rule* r *if this rule is valid on all models* $\mathcal{F}$ *then there exists an algorithm verifying if any given formula is satisfiable.*

Now we need rules in some uniform simple form, in particular — without nested temporal operations.

Definition 9. *A rule* $\mathbf{r}$ *is said to be in* reduced normal form *if* $\mathbf{r} = \varepsilon/x_1$ *where* P_f *is a finite set of propositional letters,* I *is the set (of indexes) of all agents and*

$$\varepsilon \;=\; \bigvee_{1\leqslant j\leqslant m}\Big[\bigwedge_{1\leqslant i\leqslant n} x_i^{t(j,i,0)} \wedge \bigwedge_{1\leqslant i\leqslant n}(\mathbf{N}x_i)^{t(j,i,1)} \wedge \bigwedge_{p\in P_f,j\in I} K_j p^{t(j,p,2)} \wedge \bigwedge_{1\leqslant i,k\leqslant n,l\in J}(x_i\mathbf{U}_l x_k)^{t(j,i,k,l,2)}\Big],$$

$t(j,i,0)$, $t(j,i,1)$, $t(j,p,2)$, $t(j,i,k,l,2)\in\{0,1\}$ *and, for any formula* α, $\alpha^0 := \alpha$, $\alpha^1 := \neg\alpha$.

Definition 10. *For any given rule* $\mathbf{r}$, *a rule* $\mathbf{r_{nf}}$ *in the reduced normal form is said to be a* reduced normal form of $\mathbf{r}$ *iff*

For any frame $\mathcal{F}$, *the rule* $\mathbf{r}$ *is valid in* $\mathcal{F}$ *if and only if the rule* $\mathbf{r_{nf}}$ *is valid in* $\mathcal{F}$.

Theorem 11. *There exists an algorithm running in (single) exponential time which given any rule* $\mathbf{r}$ *constructs some its reduced form* $\mathbf{r_{nf}}$.

The proofs of the similar statement for various relative relational models and rules was suggested by us quite a while ago since 1984 (eg. cf. for proof Lemma 5 in [3], or the proofs of similar statements in [18]).

The reduced normal forms of rules constructed by the algorithm from the proof of this theorem are defined uniquely.

Thus, if we are interested to investigate the problem of refutation for rules, we may restrict ourselves with consideration rules in the reduced form only.

3. Decidability problem

Now we need an elicitation of our used technique. Recall that and as it was said before, a *temporal non-transitive frame* $\mathcal{F}$ is a tuple

$$\mathcal{F} := \langle N, \{R_x^j \mid x\in N, j\in J\}, Nxt\rangle$$

such that for all $x \in N$, R_x^j is the linear order on the interval $[x, a_x^j]$ for some fixed $a_x^j \geqslant x$, $a_x^j \in N$, it might be also that R_x^j is the linear order on whole interval $[x, \infty)$, and $\forall x, y \in N$, $x\, Nxt\, y \Leftrightarrow y = x + 1$. Any model $\mathcal{M}$ based at $\mathcal{F}$ is obtained by introduction some valuation V in $\mathcal{F}$ of a set of letters. We need now to modify such models.

Definition 12. *Any model $\mathcal{M}_{Fin}$ has the following structure. For given two numbers $m, m > 1$, $n > m$,*

$$\mathcal{M}_{Fin} := \langle [0, n], \leqslant, \{R_x^j \mid x \in N, j \in J\}, \mathrm{Next}, V\rangle, \text{ where } \mathrm{Next}(n) := m + 1.$$

The relations R_x^j in such models are as follows: any R_x^j is the linear order on $[x, a_x^j]$ where (1) $x \leqslant m$ and $a_x^j \leqslant n$, or (2) $x \geqslant m$ and $a_x^j \leqslant n$ or (3) as in (2) but else R_x^j extended by the linear order on $[m+1, b^j]$, $b^j \leqslant n$, and all states from the second interval $[m+1, b^j]$ considered as strictly bigger than the states of the first one (so we do a loop). V to be just a valuation as earlier.

Now we need to clarify how to compute truth values of the formulas in such model. The rules for computation the truth values of formulas in such models w.r.t. any given valuation V are defined exactly as described earlier for usual models, simply for states x bigger than m the order R_x^j within $\leqslant$, in a sense, to be replaced by possible sequences by $Next$ and they use new R_x^j for existence solution for until.

Theorem 13. Assume that a rule r in normal reduced form is refuted in a model $\mathcal{M}$ by a valuation V, then there exists a finite model of kind $\mathcal{M}_{Fin}$ disproving r by its own valuation V (the size of such model is yet not evaluated).

Proof. Consider a model $\mathcal{M} := \langle N, \{R_x^j \mid x \in N, j \in J\}, \mathrm{Next}, V\rangle$, and assume that a rule at the reduced normal form be $r = \varepsilon / x_1$ where $\varepsilon = \bigvee_{1 \leqslant j \leqslant v} \theta_j$,

$$\theta_j = [\bigwedge_{1 \leqslant i \leqslant n} x_i^{t(j,i,0)} \wedge \bigwedge_{1 \leqslant i \leqslant n} (\mathbf{N}\ x_i)^{t(j,i,1)} \wedge \bigwedge_{1 \leqslant i,k \leqslant n, l \in J} (x_i \mathbf{U}_l x_k)^{t(j,i,k,l,2)}];\ \text{let}$$

r be refuted in a $\mathcal{M}$ by a valuation V: $\mathcal{M} \Vdash_V \neg r$.

That is all formulas from the premise of r are true at all states, but the conclusion is not true at some s, clearly we may admit that $s = 0$.

Thus for any $a \in \mathcal{F}$ there is exactly one unique θ_j which is true at a w.r.t. V, denote that θ_j by $\theta(a)$. Now we need to definite some special sets. For any $b \in \mathcal{F}$, let for any formula $\varphi := x_i \mathbf{U}_l x_j$ from the premise of the rule if $(\mathcal{M}, b) \Vdash_V x_i \mathbf{U}_l x_j$

$$Ev(\varphi, b) := \min\{k \mid b \leqslant k, bR_b^l k, (\mathcal{M}, k) \Vdash_V x_j, \forall c(b \leqslant c < k)(\mathcal{M}, k) \Vdash_V x_i\}.$$

So, $Ev(\varphi, b)$ is the minimal evidence state saying that $x_i \mathbf{U}_l x_j$ is true at b w.r.t. the view of the agent $l \in J$.

Vice versa, for any $b \in \mathcal{M}$, if $(\mathcal{M}, b) \nVdash_V x_i \mathbf{U}_l x_j$,

$$Disp(\varphi, b) := \min\{k \mid b \leqslant k, bR_b^l k, [(\mathcal{M}, k) \Vdash_V x_j \Rightarrow \exists c(b \leqslant c < k)(\mathcal{M}, c) \nVdash_V x_i]\}.$$

That is $Disp(\varphi)$ to be the minimal element disproving the formula φ w.r.t. the view of the agent $l \in J$.

Let Dm be the set of all disjunctive members of the premiss of the rule r. Sine the infinity of N there is a number m, there is a subset Dm_1 of Dm such that for any number $m_1 \geqslant m$ there is exactly one $\theta \in Dm_1$ which is true w.r.t. V at m_1 and for any θ from Dm_1 there are infinitely many numbers bigger than m at which θ is true w.r.t. V. In other worlds, the following hold

$$\forall m_1 \geqslant m \exists \theta \in Dm_1[(\mathcal{M}, m_1) \Vdash_V \theta \ \& \ [\forall \theta_1 \in Dm_1(\mathcal{M}, m_1) \Vdash_V \theta_1 \Rightarrow \theta = \theta_1]]. \tag{1}$$

$$\forall m_1 \geqslant m \forall \theta \in Dm_1[(\mathcal{M}, m_1) \Vdash_V \theta \Rightarrow \exists m_2 > (m_1 + m + ||Dm||) \ (\mathcal{M}, m_2) \Vdash_V \theta)]. \tag{2}$$

Now on, consider a smallest a where $a > m$ and $a > b$, where

$$b = max\{n + 1 \mid n \in \bigcup_{\varphi}\{Disp(\varphi, m) \cup \bigcup_{\varphi}\{Ev(\varphi, m)\} \tag{3}$$

and $\theta(m + 1) = \theta(a)$.

We will now modify our model. Let $\mathcal{M}_{Fin}$ be a model obtained form $\mathcal{M}$ as follows:

$$\mathcal{M}_{Fin} := \langle [0, m] \cup [m, a], \leqslant, \{R_x^j \mid x \in N, j \in J\}, \text{Next}, V\rangle,$$

where $\text{Next}(a) := m + 1$ and the model is defined as earlier for models of kind $\mathcal{M}_{Fin}$ and else have the following structure concerning the accessibility relations R_x^l, $x \in N$, $l \in J$: for all $x \geqslant m$, $x \in N$, if $[x, a_x^j]$ is located inside the interval $[0, a]$ in N itself we do not change R_x, otherwise

$$a_x^j := b. \tag{4}$$

We show now that the truth values for formulas from Dm in the modified model are the same as earlier.

Lemma 14. *$\forall x \in [0, a]$, and $\theta(x)$ defined in the model $\mathcal{M}$,*

$$(\mathcal{M}, x) \Vdash_V \theta(x) \Leftrightarrow (\mathcal{M}_{Fin}, c) \Vdash \theta(x).$$

We will show it using structure of the formulas $\theta(x)$. For subformulas ψ of formulas $\theta(x)$ not including operations $\mathbf{U}_l$ the statement

$$(\mathcal{M}, x) \Vdash_V \psi \Leftrightarrow (\mathcal{M}_{Fin}, c) \Vdash \psi$$

may be shown by straightforward simple induction of the length of the formulas (which hence to be omitted). For formulas $\varphi := x_i \mathbf{U}_l x_j$,

$$(\mathcal{M}, x) \Vdash_V \varphi \Leftrightarrow (\mathcal{M}_{Fin}, c) \Vdash x_i \mathbf{U}_l x_j$$

follows from our definition (3) above:

$$b = \max\{n + 1 \mid n \in \bigcup_{\varphi}\{Disp(\varphi, m) \cup \bigcup_{\varphi}\{Ev(\varphi, m)\}$$

because the presence of all evidence states and disproving states for all operations $\mathbf{U}_l$, they are all included in the modified model and that is sufficient to keep truth values of formulas of kind $x_i \mathbf{U}_l x_j$ the same. Lemma is proved. □

It concludes the proof of our theorem.

Recall that do not have yet the computable evaluation of the size of the model, and so we need to recover it.

Theorem 15. If a rule r in normal reduced form is refuted in a model $\mathcal{M}_{Fin}$ then it is refuted in some such model with a polynomial size computable from the length of the r.

Proof. Assume $\mathcal{M}_{Fin} = \langle [0,m] \cup [m,a], \ \{R^j_x \mid x \in N, j \in J\}, Nxt, V\rangle$, where Next$(a) := m+1$, $\mathbf{r} = \varepsilon/x_1$ where

$$\varepsilon = \bigvee_{1\leqslant j\leqslant m} \Big[\bigwedge_{1\leqslant i\leqslant n} x_i^{t(j,i,0)} \wedge \bigwedge_{1\leqslant i\leqslant n} (\mathbf{N}x_i)^{t(j,i,1)} \wedge \bigwedge_{1\leqslant i,k\leqslant n, l\in J} (x_i \mathbf{U}_l x_k)^{t(j,i,k,2)} \Big],$$

and $Dm(r)$ be the set of all disjunctive members of the premiss of the rule r, and, for any $x \in [0,a]$, $\theta(x)$ be the member of $Dm(r)$ which is true on x.

Now similar as in the previous lemma but in this new model consider the following definitions. For any $b \in \mathcal{M}_{Fin}$, let for any formula $\varphi := x_i \mathbf{U}_l x_j$ from the premise of the rule if $(\mathcal{M}_{Fin}, m) \Vdash_{V_j} x_i \mathbf{U}_l x_j$

$$Ev(\varphi, m) := \min\{k \mid k, mR^l_b k, k \leqslant a, (\mathcal{M}_{Fin}, k) \Vdash_V x_j,$$

$$\forall c (c \text{ between } m \text{ and } k \text{ by } R^l_b)(\mathcal{M}_{Fin}, c) \Vdash_V x_i\}.$$

So, $Ev(\varphi, m)$ is the minimal evidence state saying that $x_i \mathbf{U}_l x_j$ is true at m. Vice versa, if $(\mathcal{M}_{Fin}, m) \nVdash_V x_i \mathbf{U}_l x_j$,

$$Disp(\varphi, m) := \min\{k \mid k \leqslant a, \ mR^l_b k, [(\mathcal{M}_{Fin}, k) \Vdash_V x_j \Rightarrow$$

$$\exists c (c \text{ between } m \text{ and } k \text{ by } R^l_b)(\mathcal{M}_{Fin}, c) \nVdash_V x_i]\}.$$

That is $Disp(\varphi)$ to be the minimal element disproving the formula φ.

Let $\{a_1, \ldots, a_n\}$ be the increasing sequence of all elements from all sets $Disp(\varphi, m)$ and all $Ev(\varphi, m)$. Now on we are ready to start the rarefication procedure in order to reduce the size of the model $\mathcal{M}_{Fin}$ to a computable (from size of r) one.

STEP 1. If $a_n = a-1$ we do nothing. Otherwise consider $\theta(a-1)$ and any minimal $b \in [a_n+1, a-1]$ (if the one exists) where $\theta(a-1) = \theta(b)$. And now we delete all elements situated strictly between $b-1$ and $a-1$ and redefine relations R^l_x as follows: if a^l_x does not exceed $b-1$ or if $a^l_x \in [a-1, a]$ we let R^l_x intact. Otherwise

$$R^l_x := [x, b-1] \cup I([a-1, a^l_{a-1}]),$$

where $I([a-1, a^l_{a-1}])$ is the interval by $Next$ leading from $a-1$ to a^l_{a-1}. Let $\mathcal{M}_1$ be the model modified as shown above.

Lemma 16. *For all $x \in \mathcal{M}_1$, and $\theta(x)$ defined for $\mathcal{M}_{Fin}$*

$$(\mathcal{M}_{Fin}, x) \Vdash_{V_j} \theta(x) \Leftrightarrow (\mathcal{M}_1, x) \Vdash_{V_j} \theta(x).$$

Proof follows by straightforward computation using $\theta(a-1) = \theta(b)$ valid in $\mathcal{M}_{Fin}$.

Now we consider c, were $Next(c) = a-1$ instead of b as above and do for it the similar transformation doing proper rarefication, and, next, continue such transformation until we delete all states x with the same $\theta(x)$ moving to a_n. So, such transformation will be completed in at most $||Dm(r)||$ steps and the resulting model $\mathcal{M}_2$ by Lemma 16 will disprove r.

Now we will reduce the size of $\mathcal{M}_2$ doing rarefication within $[m, a_n]$. For this we consider separately all intervals $[a_i, a_{i+1}]$ moving down from $[a_{n-1}, a_n]$ to $[m, a_1]$.

For $[a_n-1, a_n]$ we do it as for $[b, a-1]$ above and so on. After completion this procedure we will have computable upper bound for the number of states situated between a_n and m — at most $n \times k \times ||Dm(r)|| + ||Dm(r)||$, where k is the number of all formulas of kind $x_i \mathbf{U}_l x_j$ in the rule r. Denote the obtained model by $\mathcal{M}_3$, it again will disprove r.

STEP 2. Now we will apply the same rarefication technique to the model $\mathcal{M}_3$ moving from m down towards 0, that is rarefying the interval $[0, m]$ exactly by the same procedure as for the interval $[b, a-1]$ above. Because we do not need disproving (and proving) states since we do not have a loop by $Next$ already, we need to consider only this interval itself in only one run. So, after completion this procedure we will have the model $\mathcal{M}_4$ which again will disprove r and will have size at most $n \times k \times ||Dm(r)|| + ||Dm(r)|| + k \times ||Dm(r)||$. □

Theorem 17. If a rule r in normal form is refuted in a model $\mathcal{M}_{Fin}$ then it may be refuted in some usual model $\mathcal{M}$.

Proof. We need only to apply a simple modification of the standard unraveling technique. Let $\mathcal{M}_{Fin}$ is given, it has the base set $[0, m] \cup [m, a]$, and $\text{Next}(a) := m + 1$ and $\mathbf{r} = \varepsilon/x_1$. In fact now it is sufficient to only roll the cyclic part $[m, a]$ starting from first occurrence of m in the model towards future. □

Using Lemmas 8 and Theorems 11, 13, 15 and 17 we immediately derive:

Theorem 18. *The satisfiability problem for logic TL_{NT}^{MA} is decidable.*

4. Conclusion, open problems

We immediately enumerate a set of open problem in the area. (1) To extend the obtained results on branching time logic which linear parts by operation NEXT look as frames of this paper. Similar question is answered in Rybakov [29] for frames which still within old paradigm of a kind of interval logic. (2) Study unification problem for logics this paper. The logical unification problem is impotent one because applications in AI and CS and else it may be seen as algebraic problem of finding solutions for equations in special free algebras.

That problem was in active investigation earlier for various logics (cf. Baader [1,2], Ghilardi [11,12], Rybakov [23]) and it looks interesting to find solution for our logics. (3) Study admissibly problem for them. The problem of admissibility since paper of H. Fridman [4] with the list of open logical problems was investigated for many logics (cf. eg. [15,16,18,30,31]), cf. R. Iemhoff and G. Metcalfe [5,6], cf. E. Jerabek [5,6]. But concerting nontransitive temporal linear logic the most progress was achieved only for a logic with uniform limitations on time intervals with transitivity in paper Rybakov [27]. (4) Consider the question of axiomatization for our logics.

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References

[1] F.Baader, P.Marantidis, A.Mottet, A.Okhotin, Extensions of unificationmodulo ACUI, *Mathematical Structures in Computer Science*, **30**(2019), no. 6, 1–30. DOI: 10.1017/S0960129519000185

[2] F.Baader, P.Narendran, Unification of concept terms, Proceedings of the 11th International Workshop on Unification, UNIF-97, LIFO Technical Report, 2001, 97–98.

[3] S.Babenyshev, V.Rybakov, Linear Temporal Logic LTL: Basis for Admissible Rules, *J. Logic Computation*, **21**(2011), no. 2, 157–177.

[4] H.Friedman, One Hundred and Two Problems in Mathematical Logic, *Journal of Symbolic Logic*, **40**(1975), no. 3, 113–130.

[5] E.Jerabek, Independent bases of admissible rules, *Logic Journal of the IGPL*, **16**(2008), no. 3, 249–267. DOI: 10.1093/jigpal/jzn004

[6] E.Jerabek, Bases of admissible rules of Lukasiewicz logic, *Journal of Logic and Computation*, **20**(2010), no. 6, 1149–1163. DOI: 10.1093/logcom/exp082

[7] D.M.Gabbay, I.M.Hodkinson, An axiomatization of the temporal logic with Until and Since over the real numbers. *Journal of Logic and Computation*, **1**(1990), 229–260.

[8] D.M.Gabbay, I.M.Hodkinson, M.A.Reynolds, Temporal Logic: Mathematical foundations and computational aspects. Vol. 1, Clarendon Press, Oxford, 1994.

[9] R.Goldblatt, Logics of Time and Computations, CSLI pubblications, 2nd revised and expanded edition, Chapter 6, 1992.

[10] V.Goranko, Hierarchies of Modal and Temporal Logics with Reference Pointers, *Journal of Logic, Language and Information*, **5**(1996), no. 1, 1–24.

[11] S.Ghilardi, Unification in Intuitionistic Logic, *J. of Symbolic Logic*, **64**(1999), no. 2, 859–880. DOI: 10.2307/2586506

[12] S.Ghilardi, Unification Through Projectivity. *J. of Logic and Computation*, **7**(1997), n. 6, 733–752. DOI: 10.1093/logcom/7.6.733

[13] R.Iemhoff, On the admissible rules of intuitionistic propositional logic, *Journal of Symbolic Logic*, **66**(2001), 281–294. DOI: 10.2307/2694922

[14] R.Iemhoff, G.Metcalfe, Proof theory for admissible rules, *Annals of Pure and Applied Logic*, **159**(2009), no. 1-2, 171–186. DOI: 10.1016/j.apal.2008.10.011

[15] S.Odintsov, V.Rybakov, Inference Rules in Nelson's Logics, Admissibility and Weak Admissibility, *Logica Universalis*, **9**(2015), no. 1, 93–120. DOI: 10.1007/s11787-014-0110-8

[16] S. Odintsov, V. Rybakov, Unification and admissible rules for paraconsistent minimal Johanssons' logic J and positive intuitionistic logic IPC+ , *Annals of Pure and Applied Logic*, **164**(2013), no. 7-8 771–784. DOI: 10.1016/j.apal.2013.01.001

[17] V.Rybakov, Refined common knowledge logics or logics of common information, *Archive for mathematical Logic*, **42**(2003), no. 2, 179–200. DOI: 10.1007/s001530100134

[18] V.Rybakov, Logical Consecutions in Discrete Linear Temporal Logic, *J. of Symbolic Logic*, **70**(2005), no. 4, 1137–1149. DOI: 10.2178/jsl/1129642119

[19] V.Rybakov, Linear temporal logic with until and next, logical consecutions, *Annals of Pure and Applied Logic*, **155**(2008), 32–45. DOI: 10.1016/j.apal.2008.03.001

[20] V.V.Rybakov, Logic of knowledge and discovery via interacting agents – Decision algorithm for true and satisfiable statements, *Information Sciences*, **179**(2009), no. 11, 1608–1614. DOI: 10.1016/j.ins.2008.12.008

[21] V.V.Rybakov, Linear Temporal Logic LTL_{K_n} extended by Multi-Agent Logic K_n with Interacting Agents, *Journal of logic and Computation*, **19**(2009), no. 6, 989–1017.

[22] V.Rybakov, Logical Analysis for Chance Discovery in Multi-Agents' Environment, KES 2012, Conference Proceedings, Springer, 2012, 1593–1601.

[23] V.Rybakov, Writing out unifiers in linear temporal logic, *Journal of Logic and Computation*, **22**(2012), no. 5, 1199–1206. DOI: 10.1093/logcom/exr022

[24] V.Rybakov, Non-transitive linear temporal logic and logical knowledge operations, *J. Logic and Computation*, Oxford Press, **26**(2016), no. 3, 945–958. DOI: 10.1093/logcom/exv016

[25] V.V.Rybakov, Nontransitive temporal multiagent logic, information and knowledge, deciding algorithms, *Siberian Mathematical Journal*, **58**(2017), no. 5, Pleiades Publishing (USA), 875–886. DOI: 10.1134/S0037446617050147

[26] V Rybakov, Multiagent temporal logics with multivaluations, *Siberian Mathematical Journal*, **59**(2018), no. 4, Pleiades Publishing (USA), 710–720. DOI: 10.1134/S0037446618040134

[27] V.Rybakov, Linear Temporal Logic with Non-transitive Time, Algorithms for Decidability and Verification of Admissibility, Chapter in Book: Outstanding Contributions to Logic, Spribger, 2018, 219–243.

[28] V.Rybakov, Temporal multi-valued logic with lost worlds in the past, *Sib. Elektron. Mat. Izv.*, **15**(2018), 436–449. DOI: 10.17377/semi.2018.15.039

[29] V.Rybakov, Branching time agents' logic, satisfiability problem by rules in reduced form, *Sib. Elektron. Mat. Izv.*, **16**(2019), 1158–1170. DOI: 10.33048/semi.2019.16.079

[30] R.Schmidt, D.Tishkovsky, Solving rule admissibility problem for S4 by a tableau method, Proceeding, Automated Reasoning Workshop, 2011, 30–31.

[31] R.Schmidt, D.Tishkovsky, S.Babenyshev, V.Rybakov, A tableau method for checking rule admissibility in S4, *Electronic Notes in Theoretical Computer Science*, **262**(2010), 17–32. DOI: 10.1016/j.entcs.2010.04.003

[32] J. van Benthem, Tense logic and time, *Notre Dame J. Formal Logic*, **25**(1984), no. 1, 1–16.

[33] M.Y.Vardi, An automata-theoretic approach to linear temporal logic, In: Banff Higher Order Workshop, 1995, 238–266.
Available at http://citeseer.ist.psu.edu/vardi96automatatheoretic.html.

[34] M.Y.Vardi, Reasoning about the past with two-way automata, In: Larsen K.G., Skyum S., Winskel G., editors. ICALP, LNCS, Springer, vol. 1443, 1998, 628–641.
DOI: 10.1007/BFb0055090

[35] Y.Venema, Temporal Logic, Chapter 10, In: L. Goble (ed), Blackwell Guide on Philosophical Logic, Blackwell Publishers, 2001, 203–223.

Мультиагентная временная логика с операциями статистического знания агентов

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Аннотация. Исследуется мультиагентная временная логика с нестандартными нетранзитивными отношениями временной достижимости и операциями статического знания агентов. Находится алгоритм, решающий проблему выполнимости и разрешимости.

Ключевые слова: временная логика, мультиагентность, неклассические логики, информация, представление знаний, разрешающие алгоритмы, разрешимость, выполнимость.