TEMPORAL LOGIC WITH OVERLAP TEMPORAL RELATIONS GENERATED BY TIME STATES THEMSELVES

ABSTRACT. The paper studies temporal logic with non-standard temporal accessibility relations. The logic is generated by semantic underground models, and any model has as a base a frame with temporal relations generated by temporal states themselves, – potentially any state possesses by its own temporal accessibility relation, possibly all different ones. We see it as most plausible modelling, because any time state has, in principle, its own view on what is past (or future). Time relations may have non-empty overlaps, so to be totally intransitive and, thus that approach to be suitable to analyze most general cases of reasoning about computation, passing information its reliability and other areas in AI and CS. The main mathematical question under consideration is existence of algorithms for solving satisfiability problems. We solve this problem and find the algorithms. In the final part of our paper we set interesting open problems.

Keywords: temporal logic, non-classical logics, information, knowledge representation, deciding algorithms, computability, information, satisfiability, decidability.

1. INTRODUCTION

This paper combines two issues for interest: (1) pure mathematical one consists in construction mathematica models for flaw of time and transition of information by Kripke-Hintikka like relational models and logics syntaxis – formulas and other syntactical instruments (to analyze laws and properties of such models), and (2) possible applications to passing information, analyses of correctness the reasoning, plausibility, getting hidden infirmation, consistency and reliability of knowledge and other areas of AI and CS. The concept of knowledge, which maybe at first glance looking as a kind of a stable, correct, profoundly verified (and supported?)

information is in the centre of research in CS and Philosophy. It and related to it areas use instruments of temporal logic for representation and development computational tools.

As in general, the concept of knowledge in terms of symbolic logic, probably, may be dated to the end of 1950. At 1962 Hintikka wrote the book: *Knowledge and Belief*, the first book-length work to suggest using modalities to capture the semantics of knowledge. This book laid much of the groundwork for the subject, but a great deal of research has taken place since that time. Temporal logic since then got to be a popular area in mathematical symbolic logic and CS; a lot of impressive results where obtain (cf. for historical outlook Gabbay, Hodkinson, Reynolds[5, 6], Goldblat [7], Goranko [8], van Benthem [28], Yde Venema [31]).

Since invention of the linear temporal logic $\mathcal{LTL}$ with operation U - until - by Amir Pnueli that system was investigate from many viewpoints due to interesting mathematical representation and useful applications to analysis of protocols for computations, verification of consistency. Automaton technique to solve satisfiability in this logic was developed by Vardi [29, 30]). From reasonably modern results concerning this logic I would mention the solution for admissibility problem for $\mathcal{LTL}$ in Rybakov [14, 15], the basis for admissible rules of $\mathcal{LTL}$ was obtained in Babenyshev and Rybakov [3]. The unification problem for $\mathcal{LTL}$ was solved in [19]. Concerning applications of logical methods in AI and CS, the tools around temporal logic work well for analysis in multi-agent environment (cf. eg. [16, 17]).

In current time temporal logic was investigated from many viewpoints, in particular extensions of $\mathcal{LTL}$ for the case of non-transitive models, were studied in Rybakov [20, 21, 25] for the case of the interval versions of the logic. Also modelling multi-agent reasoning via temporal models was applied in Rybakov [18, 22, 24] for the versions of liner logic.

This paper is devoted to study an importance modification of $\mathcal{LTL}$ – a logic based on non-transitive time with possible time overlaps on temporal accessibility relations - so by nature intransitive one. But most innovative part here is that the temporal relations on generating models are individual for any time state. This looks as quite a new approach not touched yet in literature and most plausible for real simulations of time runs. Besides the author, as mentioned above, actively studied non-transitive temporal logics generated over linear time. I was trying to resolve the most general case - when the temporal relations may be unbound and else not placed all in infinite sequences of fixed intervals of time, since this limitation looked as a bit artificial one, but I was unsuccessful. Here we found the solution and yet via new approach which successfully packed all limitations in one new approach which successfully allowed to resolve the problem. The main mathematical problem under we study is existence of algorithms for solving the satisfiability problems. We solve this problem and find the algorithms via reduction the problem to special models with computable size. The paper is concluded with setting interesting open problems.

2. Logical Language, Models with Overlap Relations

As we noted above, the most innovative point in the paper is usage of temporal accessibility relations separate, so to say, individual, for any temporal state. That looks as we have infinite number of accessibility relations, and it seems as we will

need infinite set of temporal operations in the logical language. But, in fact, it is not a case, and we can model this approach in usual temporal language.

So the logical language consists of potentially infinite set of propositional letters P, Boolean logical operations, operation $\mathcal{N}$ (next), operation U (until). The formation rules for compound formulas are as always: any letter from P is a formula; the set of all formulas is closed w.r.t. applications of Boolean logical operations, the unary operation $\mathcal{N}$ (next) and the binary operation U (until); $\varphi U \psi$ to be read φ holds until ψ will be true, $\mathcal{N}\varphi$ says φ is true in next temporal state.

To model temporal flow we will use new modified Kripke-Hintikka like models based on linear order at natural numbers.

Definition 1. *A* linear temporal non-transitive frame *is a tuple*

$$\mathcal{F} := \langle N, \{R_x \mid x \in N\}, Nxt, \rangle, \text{ such that}$$

for all $x \in N$, R_x is the linear order on the interval $[x, a_x]$ for some $a_x \geq x, a_x \in N$, it might be also that R_a is the linear order on whole interval $[a, \infty]$. $\forall x, y \in N$, $x \ Nxt \ y \Leftrightarrow y = x + 1$.

It may happen that xR_xa_x, $y \in (x, a_x)$ and $not(yR_ya_x)$. So to say y is a state situated earlier than x but y remember even less as x remember. Besides it is clear that in total the all relations form non-transitive relation: it may happen xR_xa_x, $x < y < a_x$, so (aR_xy), (yR_ya_y) but $not(xR_ya_y)$.

A model $\mathcal{M}$ on any $\mathcal{F}$ to be defined by introduction a valuation V on $\mathcal{F}$: for a set of propositional letters p: $V(p) \subseteq N$, and V is extended to all formulas as follows:

Definition 2. *For any $a \in N$:*

$$(N, a) \Vdash_V p \quad \Leftrightarrow \quad p \in V(p);$$
$$(N, a) \Vdash_V \neg\varphi \quad \Leftrightarrow \quad (N, a) \nVdash_V \varphi;$$
$$(N, a) \Vdash_V (\varphi \ \wedge \ \psi) \quad \Leftrightarrow \quad ((N, a) \Vdash_V \varphi) \wedge ((N, a), \Vdash_V \psi);$$
$$(N, a) \Vdash_V (\varphi \ \vee \ \psi) \quad \Leftrightarrow \quad ((N, a) \Vdash_V \varphi) \vee ((N, a) \Vdash_V \psi);$$
$$(N, a) \Vdash_V (\varphi \ \rightarrow \ \psi) \quad \Leftrightarrow \quad ((N, a) \Vdash_V \psi) \vee ((N, a) \nVdash_V \psi);$$

for formulas of sort $\varphi U \psi$ we define the truth values as follows:

$$(N, c) \Vdash_V (\varphi \ U \ \psi) \quad \Leftrightarrow$$
$$\exists b \in N[(cR_cb) \wedge ((N, b) \Vdash_V \psi) \wedge$$
$$\forall y[(y \geq b, \& y < b) \Rightarrow (N, y) \Vdash_V \varphi]];$$

$$(N, a) \Vdash_V \mathcal{N}\varphi \quad \Leftrightarrow \quad [(a \ Nxt \ b) \Rightarrow (N, b) \Vdash_V \varphi].$$

$(N, a) \Vdash_V \varphi$ to be read *the formula φ is true (valid)* at the state a w.r.t. the valuation V. We see that the truth of any formula with main temporal operation U at a state a refers only to the unique accessibility relation R_a for a. Sometimes we will use notation $Nxt(a) = b$ or $Next(a) = b$ to say that $a \ Nxt \ b$.

Definition 3. *The logic T_L^{Ov} is the set of all formulas which are valid at any state of any model based at any temporal frame $\mathcal{F}$.*

General illustrations why time flow may be seen as non-transitive and what might be usage of such approach given in Rybakov [20, 21, 22, 24, 25].

3. A Technique via Reduced Forms

Our aim is to show that the satisfiability problem for introduced logic is decidable. Usual technique based at filtration, usage temporal degree of formulas and dropping points do not work for this semantics since the relations as total non-transitive and rules for computation truth values of formulas with U are different from standard. We will use a modification of our old technique for reduction of formulas to rules (which we have already used earlier many times for different purposes (cf. e. g. [17, 15]) and transformation the latter ones to so-called reduced forms. We now briefly recall this technique.

A *rule* is an expression $\mathbf{r} := \varphi_1(x_1, \ldots, x_n), \ldots, \varphi_s(x_1, \ldots, x_n) \, / \, \psi(x_1, \ldots, x_n)$, where all $\varphi_k(x_1, \ldots, x_n)$ and $\psi(x_1, \ldots, x_n)$ are formulas constructed out of letters (variables) $x_1, \ldots, x_n$.

Formulas $\varphi_k(x_1, \ldots, x_n)$ are called *premises* and $\psi(x_1, \ldots, x_n)$ is the *conclusion*. The rule $\mathbf{r}$ means that $\psi(x_1, \ldots, x_n)$ (conclusion) follows (logically follows) from the assumptions $\varphi_1(x_1, \ldots, x_n), \ldots, \varphi_s(x_1, \ldots, x_n)$. The definition of the validness of a rule is the same for any relational model. To recall it, assume that a model $\mathcal{M}$ and a rule $\mathbf{r}$ are given.

Definition 4. *The rule* $\mathbf{r} := \varphi_1(x_1, \ldots, x_n), \ldots, \varphi_s(x_1, \ldots, x_n) \, / \, \psi(x_1, \ldots, x_n)$, *is valid on the model* $\mathcal{M}$ *based at a frame* $\mathcal{F}$ *iff*

$$\left[\forall a \left((\mathcal{F}, a) \Vdash_V \bigwedge_{1 \leq i \leq s} \varphi_i \right) \right] \;\; \Rightarrow \;\; \left[\forall a \left((\mathcal{F}, a) \Vdash_V \psi \right) \right].$$

If $\forall a \left((\mathcal{F}, a) \Vdash_V \bigwedge_{1 \leq i \leq s} \varphi_i \right)$ *but* $\exists a \left((\mathcal{F}, a) \nVdash_V \psi \right)$, *then we say that* $\mathbf{r}$ *is refuted in* $\mathcal{F}$ *by* V *and we denote this fact as* $\mathcal{F} \nVdash_V \mathbf{r}$.

Definition 5. *A rule* $\mathbf{r}$ *is valid (or true) on a frame* $\mathcal{F}$ *iff* $\mathbf{r}$ *is true (valid) on any model based on* $\mathcal{F}$.

Definition 6. *A formula* φ *is satisfiable iff there is a frame* $\mathcal{F}$ *and a valuation* V *on* $\mathcal{F}$ *such that* φ *is true w.r.t.* V *at same sate from* $\mathcal{F}$

Lemma 1. *For a formula* φ, φ *is satisfiable iff the rule* $x \to x/\neg\varphi$ *may be refuted in some model* $\mathcal{M}$.

The proof is obvious - immediately follows from definitions. Thus we have

Lemma 2. *If there is an algorithm verifying for any given rule* r *if this rule is valid on all models* $\mathcal{F}$ *then there exists an algorithm verifying if any given formula is satisfiable.*

Now we need rules in some uniform simple form, in particular — without nested temporal operations.

Definition 7. *A rule* $\mathbf{r}$ *is said to be in* reduced normal form *if* $\mathbf{r} = \varepsilon/x_1$ *where*

$$\varepsilon \;\; = \;\; \bigvee_{1 \leq j \leq m} \Big[\bigwedge_{1 \leq i \leq n} x_i^{t(j,i,0)} \wedge \bigwedge_{1 \leq i \leq n} (N x_i)^{t(j,i,1)} \wedge$$
$$\wedge \bigwedge_{1 \leq i,k \leq n} (x_i \, U \, x_k)^{t(j,i,k,2)} \Big],$$

$t(j,i,0)$, $t(j,i,1)$, $t(j,i,k,2) \in \{0,1\}$ *and, for any formula* α, $\alpha^0 := \alpha$, $\alpha^1 := \neg\alpha$.

Definition 8. *For any given rule* **r**, *a rule* **r$_{nf}$** *in the reduced normal form is said to be a* reduced normal form *of* **r** *iff*

> *For any frame $\mathcal{F}$, the rule* **r** *is valid in $\mathcal{F}$ if and only if the rule* **r$_{nf}$** *is valid in $\mathcal{F}$.*

Theorem 1. *There exists an algorithm running in (single) exponential time which given any rule* **r** *constructs some its reduced form* **r$_{nf}$**.

Proof. The proofs of the similar statement for various relative relational models and rules was suggested by us quite a while ago since 1984 (eg. cf. Lemma 5 in [3], or the proofs of similar statements in [14]).

The reduced normal forms of rules constructed by the algorithm from the proof of this theorem are defined uniquely.

Thus, if we are interested to investigate the problem of refutation for rules, we may restrict ourselves with consideration rules in the reduced form only.

4. Main Proofs, Results

First we need special auxiliary models. Recall that a *linear temporal non-transitive frame $\mathcal{F}$* is a tuple $\mathcal{F} := \langle N, \{R_x \mid x \in N\}, Nxt, \rangle$, such that for any $x \in N$, R_x is the linear order on the interval $[x, a_x]$ for some a_x chosen for each x, it might be also that R_x is the linear order on whole interval $[x, \infty)$. A model $\mathcal{M}$ based at $\mathcal{F}$ is obtained by introduction some valuation V in $\mathcal{F}$ of a set of letters.

Definition 9. Any $\mathcal{M}_{+Lp}$ model has the following structure. For $m, m > 1, n > m$, $\mathcal{M}_{+Lp} = \langle [0, n], \leq, \text{Next}, V \rangle$ where $\text{Next}(n) := m + 1$.

The relations R_x in such models are as follows: any R_x is the linear order on $[x, a_x]$ where (1) $x \leq m$ and $a_x \leq n$, or (2) $x \geq m$ and $a_x \leq n$ or (3) as in (2) but else R_x extended by the linear order on $[m + 1, b]$, $b \leq n$, and all elements from the second interval $[m + 1, b]$ considered as strictly bigger than the states of the first one (so we do a loop). V to be just a valuation as earlier.

The rules for computation the truth values of formulas in such models w.r.t. any given valuation V are defined exactly as described earlier for usual models, simply for states x bigger than m the order R_x within $\leq$, in a sense, to be replaced by possible sequences by $Next$ and they use new R_x for existence solution for until.

Theorem 2. If a rule r in normal reduced form is refuted in a model $\mathcal{M}$ by a valuation V, then there exists a finite model of kind $\mathcal{M}_{+Lp}$ disproving r by its own valuation V (the size of such model is not identified yet).

Proof. Let $\mathcal{M} := \langle N, \{R_x \mid x \in N\}, Nxt, V \rangle$, and the rule an the reduced normal form is $r = \varepsilon/x_1$ where $\varepsilon = \bigvee_{1 \leq j \leq v} \theta_j$,

$$\theta_j = [\bigwedge_{1 \leq i \leq n} x_i^{t(j,i,0)} \wedge \bigwedge_{1 \leq i \leq n} (N\ x_i)^{t(j,i,1)} \wedge \bigwedge_{1 \leq i,k \leq n} (x_i U x_k)^{t(j,i,k,2)}]; \text{ let}$$

r be refuted in a $\mathcal{M}$ by a valuation V: $\mathcal{M} \Vdash_V \neg r$. That is all formulas from the premise of r are true at all states, but the conclusion is not true at some s, clearly we may admit that $s = 0$.

Thus for any $a \in \mathcal{F}$ there is exactly one unique θ_j which is true at a w.r.t. V, denote that θ_j by $\theta(a)$. Now we need to definite some special sets. For any $b \in \mathcal{F}$, let for any formula $\varphi := x_i U x_j$ from the premise of the rule if $(\mathcal{M}, b) \Vdash_{V_j} x_i U x_j$

$$Ev(\varphi, b) := min\{k \mid b \leq k, bR_b k, (\mathcal{M}, k) \Vdash_V x_j, \forall c(b \leq c < k)(\mathcal{M}, k) \Vdash_V x_i\}.$$

So, $Ev(\varphi, b)$ is the minimal evidence state saying that $x_i U x_j$ is true at b. Vice versa, for any $b \in \mathcal{M}$, if $(\mathcal{M}, b) \nVdash_{V_j} x_i U x_j$,

$$Disp(\varphi, b) := min\{k \mid b \leq k, bR_b k, [(\mathcal{M}, k) \Vdash_V x_j \Rightarrow \exists c(b \leq c < k)(\mathcal{M}, c) \nVdash_V x_i]\}.$$

That is $Disp(\varphi)$ to be the minimal element disproving the formula φ.

Let Dm be the set of all disjunctive members of the premiss of the rule r. Sine the infinity of N there is a number m, there is a subset Dm_1 of Dm such that for any number $m_1 \geq m$ there is exactly one $\theta \in Dm_1$ which is true w.r.t. V at m_1 and for any θ from Dm_1 there are infinitely many numbers bigger than m at which θ is true w.r.t. V. In other worlds, the following hold

(1)
$$\forall m_1 \geq m \exists \theta \in Dm_1[(\mathcal{M}, m_1) \Vdash_V \theta \&$$

$$[\forall \theta_1 \in Dm_1(\mathcal{M}, m_1) \Vdash_V \theta_1 \Rightarrow \theta = \theta_1]].$$

(2)
$$\forall m_1 \geq m \forall \theta \in Dm_1[(\mathcal{M}, m_1) \Vdash_V \theta \Rightarrow$$

.

$$\exists m_2 > (m_1 + m + ||Dm||) (\mathcal{M}, m_2) \Vdash_V \theta)].$$

Now on, consider a smallest a where $a > m$ and $a > b$, where

(3)
$$b = max\{n + 1 \mid n \in \bigcup_{\varphi}\{Disp(\varphi, m) \cup \bigcup_{\varphi}\{Ev(\varphi, m)\}$$

and $\theta(m + 1) = \theta(a)$.

We will now modify our model. Let $\mathcal{M}_{+Lp}$ be a model obtained form $\mathcal{M}$ as follows:

$$\mathcal{M}_{+Lp} = \langle [0, m] \cup [m, a] \rangle,$$

where $Next(a) := m + 1$ and the model is defined as earlier for models of kind $\mathcal{M}_{+Lp}$ and else have the following structure concerning the accessibility relations $R_x, x \in N$.

For all $x \geq m, x \in N$, if $[x, a_x]$ is located inside $[0, a]$ we do not change R_x. otherwise

(4)
$$a_x := b$$

.

We show now that the truth values for formulas from Dm in the modified model are the same as earlier.

Lemma 3. *For any $x \in [0, a]$, and $\theta(x)$ defined in the model $\mathcal{M}$,*

$$(\mathcal{M}, x) \Vdash_V \theta(x) \Leftrightarrow (\mathcal{M}_{+Lp}, c) \Vdash \theta(x).$$

Proof goes by structure of formulas $\theta(x)$. For components of such formulas not including operations U the similar statement to be shown by straightforward simple induction of the length of the formulas. For formulas $\varphi := x_i U x_j$,

$$(\mathcal{M}, x) \Vdash_V \Leftrightarrow (\mathcal{M}_{+Lp}, c) \Vdash x_i U x_j$$

follows from our definition (3) above:

$$b = max\{n + 1 \mid n \in \bigcup_{\varphi}\{Disp(\varphi, m) \cup \bigcup_{\varphi}\{Ev(\varphi, m)\}$$

because the presence of all evidence states and disproving states for operation U, they are all included in the modified model and that is sufficiennnt to keep truth values of formulas of kind $x_i U x_j$ the same $\square$. Lemma is proved.

It concludes the proof of our theorem.

Now we need to find (compute) an upper bound for size of finite models refuting the rules.

Theorem 3. If a rule r in normal reduced form is refuted in a model $\mathcal{M}_{+Lp}$ then it is refuted in some such model with a polynomial size computable from the length of the r.

Proof. Let $\mathcal{M}_{+Lp} := \langle [0, m] \cup [m, a], \leq, \text{Next}, V \rangle$, where $\text{Next}(n) := m + 1$, $\mathbf{r} = \varepsilon/x_1$ where

$$\varepsilon = \bigvee_{1 \leq j \leq m} \left[\bigwedge_{1 \leq i \leq n} x_i^{t(j,i,0)} \wedge \bigwedge_{1 \leq i \leq n} (Nx_i)^{t(j,i,1)} \wedge \right.$$
$$\left. \wedge \bigwedge_{1 \leq i,k \leq n} (x_i U x_k)^{t(j,i,k,2)} \right],$$

and $Dm(r)$ be the set of all disjunctive members of the premiss of the rule r, and, for any $x \in [0, a]$, $\theta(x)$ be the member of $Dm(r)$ which is true on x.

Now similar as in the previous lemma but in this new model consider the following definitions. Consider the chosen branching state $m \in \mathcal{M}_{+Lp}$; let for any formula $\varphi := x_i U x_j$ from the premise of the rule if $(\mathcal{M}_{+Lp}, m) \Vdash_{V_j} x_i U x_j$

c between m and k by R_b^l)

$$Ev(\varphi, m) := min\{k \mid k, mR_bk, k \leq a, (\mathcal{M}_{+Lp}, k) \Vdash_V x_j,$$

$$\forall c(b \leq c)(\mathcal{M}, k) \Vdash_V x_i\}.$$

So, $Ev(\varphi, m)$ is the minimal evidence state saying that $x_i U x_j$ is true at m. Vice versa, if $(\mathcal{M}_{+Lp}, m) \nVdash_{V_j} x_i U x_j$,

$$Disp(\varphi, m) := min\{k \mid, k, k \leq a, \ mR_bk, [(\mathcal{M}_{+Lp}k) \Vdash_V x_j \Rightarrow$$

$$\exists c(b \leq c < k)(\mathcal{M}_{+Lp}, c) \nVdash_V x_i]\}.$$

That is $Disp(\varphi)$ to be the minimal element disproving the formula φ.

Let $\{a_1, \ldots, a_n\}$ be the increasing sequence of all elements from all sets $Disp(\varphi, m)$ and all $Ev(\varphi, m)$. Now on we are ready to start the rarefication procedure in order to reduce the size of the model $\mathcal{M}_{+Lp}$ to a computable (from size of r) one.

STEP 1. If $a_n = a - 1$ we do nothing. Otherwise consider $\theta(a - 1)$ and any minimal $b \in [a_n + 1, a - 1]$ (if the one exists) where $\theta(a - 1) = \theta(b)$. And now we delete all elements situated strictly between $a - 1$ and $b - 1$ and redefine relations R_x as follows: if a_x does not exceed $b - 1$ or if $a_x \geq a_1$ we let R_x intact. Otherwise

$$R_x := [x, b - 1] \cup [a - 1, a_{a-1}].$$

Let $\mathcal{M}_1$ be the model modified as shown above.

Lemma 4. *For all $x \in \mathcal{M}_1$, and $\theta(x)$ defined for $\mathcal{M}_{+Lp}$*

$$(\mathcal{M}_{+Lp}, x) \Vdash_{V_j} \theta(x) \Leftrightarrow (\mathcal{M}_1, x) \Vdash_{V_j} \theta(x).$$

Proof follows by straightforward computation using $\theta(a - 1) = \theta(b)$ valid in $\mathcal{M}_{+Lp}$ $\square$

Now we consider c, were $Next(c) = a - 1$ instead of b as above and do for it the similar transformation doing proper rarefication, and, next, continue such transformation until we delete all states x with the same $\theta(x)$ moving to a_n. So, such transformation will be completed in at most $||Dm(r)||$ steps and the resulting model $\mathcal{M}_2$ by Lemma 4 will disprove r.

Now we will reduce the size of $\mathcal{M}_2$ doing rarefication within $[m, a_n]$. For this we consider separately all intervals $[a_i, a_i + 1]$ moving down from $[a_n - 1, a_n]$ to $[m, a_1]$.

For $[a_n - 1, a_n]$ we do it as for $[b, a - 1]$ above and so on. After completion this procedure we will have computable upper bound for the number of states situated between a_n and m - at most $n \times k \times ||Dm(r)|| + ||Dm(r)||$, where k is the number of all formulas of kind $x_i U x_j$ in the rule r. Denote the obtained model by $\mathcal{M}_3$, it again will disprove r.

STEP 2. Now we will apply the same rarefication technique to the model $\mathcal{M}_3$ moving from m down towards 0, that is rarefying the interval $[0, m]$ exactly by the same procedure as for the interval $[b, a - 1]$ above. Because we do not need disproving (and proving) sates since we do not have a loop by $Next$ already, we need to consider only this interval itself in only one run. So, after completion this procedure we will have the model $\mathcal{M}_4$ which again will disprove r and will have size at most $n \times k \times ||Dm(r)|| + ||Dm(r)|| + k \times ||Dm(r)||$. $\square$

Theorem 4. If a rule r in normal form is refuted in a model $\mathcal{M}_{+Lp}$ then it may be refuted in some usual model $\mathcal{M}$.

Proof. We need only to apply a simple modification of the standard unraveling technique. Let $\mathcal{M}_{+Lp}$ is based at the st $[0, m] \cup [m, a]$, where $Next(a) := m + 1$, $\mathbf{r} = \varepsilon/x_1$. In fact now it is sufficient to only roll the cyclic part $[m, a]$ starting from first occurrence of m in the model towards future.

Using Lemmas 1,2 and Theorems 1,2,3 and 4 we immediately derive:

Theorem 5. *The satisfiability problem for T_L^{Ov} is decidable. The logic T_L^{Ov} itself is decidable*

Notice that we may consider the reduced version of this logic T_L^{Ov} - the one $T_L^{Ov-Next}$ without the logical operation $\mathcal{N}$-next. Since we did not use this operation ever in our proofs the following theorem holds.

Theorem 6. *The satisfiability problem for $T_L^{Ov-Next}$ is decidable. The logic $T_L^{Ov-Next}$ itself is decidable*

We think there are several open problems concerning this research. (1) To extend the obtained results on branching time logic which linear parts by operation NEXT look as frames of this paper. Similar question is answered in Rybakov [25] for frames which still within old paradigm of a kind of interval logic. (2) Study problem of unification for studied in our paper logics. The logical unification problem is impotent one as applications in AI and CS and may be seen as algebraic problem of finding solutions for equations in free algebras. That problem was in active investigation earlier (cf. Baader [1, 2], Ghilardi [9, 10], Rybakov [19]) and it looks very attractive to find solution for our introduced logic. (3) Study admissibly problem for it. The problem of admissibility since paper of H.Fridman [4] with the list of open logical problems was investigated for many logics (cf. eg. [26, 27, 14, 11, 12]). But concerting nontransitive temporal linear logic the most progress was achieved only for a logic with uniform limitations on time intervals with transitivity in paper Rybakov [23]. (4) Consider the question of axiomatization for our logic. (4) Embed the agents' logic components in the invented temporal logic.

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