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On Creeping Motion of Binary Mixture in a Tube with Rectangular Cross-Section

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The problem of motion of a binary mixture in a tube with rectangular cross-section is considered in the paper. Exact stationary and non-stationary solutions are obtained. Solutions are presented in the form of series. It is proved that the solution reaches a stationary state with increasing time.

Keywords: binary mixture, stationary problem, non-stationary problem, creeping motion.

Introduction

In fluid mechanics one can distinguish the classical models which include equations of gas dynamics, Navier-Stokes equation, etc. At the present time interest is aroused in non-classical hydrodynamic models. The model of convection with the effect of thermal diffusion is a non-classical hydrodynamic model. This complicated model describes accurately actual physical processes. Thus it is necessary to study various submodels of this complicated model. Solutions of stationary and non-stationary problems on the motion of binary mixture in a horizontal cylindrical tube have been obtained in [1]. In this paper we study the motion of binary mixture in a tube with rectangular cross-section. The exact solutions of stationary and non-stationary problems are obtained. The exact solutions have always been of considerable importance in correct understanding of various phenomena. They are used as "test problems" to verify correctness of various approximations and to estimate the accuracy of numerical methods.

1. Problem statement

Let us assume that the motion of mixture is described by the following system of equations [2, 3]

$$\begin{aligned} \mathbf{u}_t + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\frac{1}{\rho_0} \nabla \bar{p} + \nu \nabla^2 \mathbf{u} + \mathbf{g}(\beta_1 T + \beta_2 C); \\ T_t + \mathbf{u} \cdot \nabla T &= \chi \nabla^2 T; \\ C_t + \mathbf{u} \cdot \nabla C &= D \nabla^2 C + \alpha D \nabla^2 T; \\ \nabla \cdot \mathbf{u} &= 0, \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the fluid velocity vector; $\bar{p}$ is the pressure deviation from hydrostatic pressure; ν is the kinematic viscosity coefficient; $\mathbf{g}$ is the vector of gravitational acceleration; χ is the thermal

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diffusivity coefficient; D is the diffusion coefficient; α is the parameter of thermal diffusion, ρ_0 is the mass density.

System (1) admits the following operator $-\partial_z + \rho_0 g x (\beta_1 A + \beta_2 B) \partial_{\bar{p}} + A \partial_T + B \partial_C$ where $A > 0$ and B are constants. Then invariant solutions of this problem have the following form [1]

$$\begin{aligned} \mathbf{u} &= (u(t, x, y); v(t, x, y); w(t, x, y)); \\ \bar{p} &= -(\beta_1 A + \beta_2 B) g \rho_0 x z + q(t, x, y); \\ T &= -Az + \theta(t, x, y), \quad C = -Bz + c(t, x, y), \end{aligned} \quad (2)$$

and functions u, v, w, q, θ, c satisfy the system of differential equations with three independent variables x, y, t .

In system (1) with conditions (2) we introduce the dimensionless variables

$$\begin{aligned} u &= \frac{\nu}{h} Pr G^2 \tilde{u}, \quad v = \frac{\nu}{h} Pr G^2 \tilde{v}, \quad w = \frac{\nu}{h} Pr G \tilde{w}, \quad q = \rho_0 \beta_1 g A h^2 Pr G \tilde{q}, \\ \theta &= A h Pr G \tilde{\theta}, \quad c = \frac{A \beta_1 h Pr G \tilde{c}}{\beta_2}, \end{aligned} \quad (3)$$

where h is the characteristic size, G is the Grashof number, Pr is the Prandtl number, S is the Schmidt number, ε_1 and ε are thermal diffusion parameters. These dimensionless variables satisfy the following relations

$$G = \frac{A \beta_1 g h^4}{\nu^2}, \quad Pr = \frac{\nu}{\chi}, \quad S = \frac{\nu}{D}, \quad \varepsilon = -\frac{\alpha \beta_2}{\beta_1}, \quad \varepsilon_1 = \frac{\chi \beta_2 B}{D \beta_1 A}.$$

Substituting (2) and (3) into (1), we obtain the system of equation (“wave” symbol is omitted for convenience)

$$\begin{aligned} u_t + \lambda(uu_x + vv_y) &= -q_x + \Delta_2 u + \theta + c; \\ v_t + \lambda(uv_x + vv_y) &= -q_y + \Delta_2 v; \\ w_t + \lambda(uw_x + vw_y) &= -x + \Delta_2 w; \\ u_x + v_y &= 0, \quad Pr \theta_t + \lambda Pr(u\theta_x + v\theta_y) - w = \Delta_2 \theta; \\ Sc c_t + \lambda Sc(uc_x + vc_y) - \varepsilon_1 w &= \Delta_2 c - \varepsilon \Delta_2 \theta, \end{aligned} \quad (4)$$

where $\lambda = Pr G^2$; Δ_2 is the Laplace operator, $\Delta_2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$.

Further we consider the creeping motion of the binary mixture in a tube with rectangular cross-section when $\lambda = 0$ in system (4). Geometry of the flow is shown in Fig. 1.

System of equation (4) is transformed into the system

$$\begin{aligned} u_t &= -q_x + \Delta_2 u + \theta + c, \quad v_t = -q_y + \Delta_2 v; \\ u_x + v_y &= 0; \end{aligned} \quad (5)$$

$$w_t = -x + \Delta_2 w; \quad (6)$$

$$Pr \theta_t - w = \Delta_2 \theta; \quad (7)$$

$$Sc c_t - \varepsilon_1 w = \Delta_2 c - \varepsilon \Delta_2 \theta. \quad (8)$$

The viscous friction force arises when fluid moves relative to a body at rest. This force is directed oppositely to the velocity of motion. This is phenomenon of tangential forces that hinder

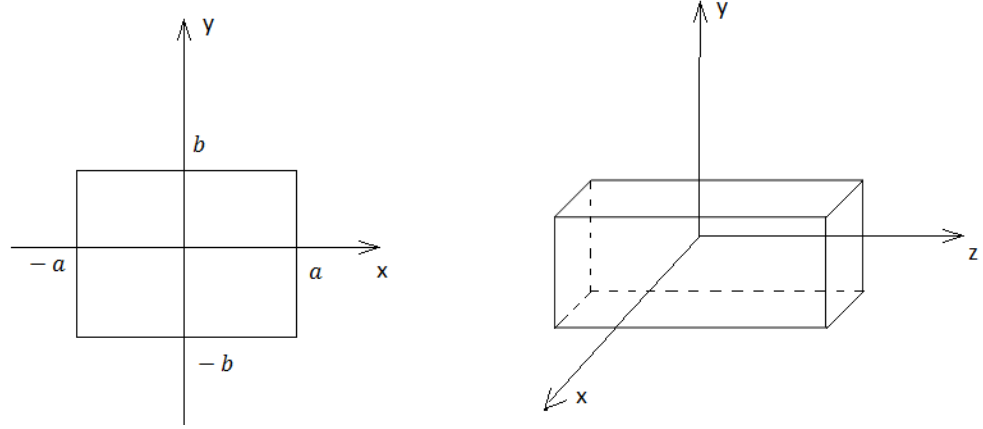


Fig. 1 Geometry of the flow

the movement of fluid near solid wall. On the boundary $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$ we set shear stresses [4]

$$\left. \frac{\partial \mathbf{u}}{\partial \tau_j} \right|_{\Gamma} = 0,$$

where $\Gamma_1 : x = a$, $\Gamma_2 : y = b$, $\Gamma_3 : x = -a$, $\Gamma_4 : y = -b$ are solid walls.

Also on solid walls we set temperature

$$T|_{\Gamma} = T_0$$

and the mass flux through the boundary is zero:

$$\left(\frac{\partial c}{\partial n} + D_T \frac{\partial T}{\partial n} \right) \Big|_{\Gamma} = 0,$$

where D_T is the thermal-diffusion coefficient, $\alpha = -D_T/T_0 D$, and T_0 is the characteristic temperature.

Let us take $h = a$ then on Γ_1 we get $x = 1$; on Γ_2 : $y = ba^{-1} = \delta$; on Γ_3 : $x = -1$; on Γ_4 : $y = -\delta$. We have the following boundary conditions:

$$\begin{aligned} u_y(t, 1, y) = u_y(t, -1, y) = u_y(t, x, \delta) = u_y(t, x, -\delta) = 0, \\ v_x(t, 1, y) = v_x(t, -1, y) = v_x(t, x, \delta) = v_x(t, x, -\delta) = 0; \end{aligned} \quad (9)$$

$$w(t, 1, y) = w(t, -1, y) = w(t, x, \delta) = w(t, x, -\delta) = 0; \quad (10)$$

$$\begin{aligned} \theta(t, 1, y) = \theta_1^1(t, y), \theta(t, -1, y) = \theta_2^1(t, y), \\ \theta(t, x, \delta) = \theta_1^2(t, x), \theta(t, x, -\delta) = \theta_2^2(t, x); \end{aligned} \quad (11)$$

$$(c_x - \varepsilon \theta_x)|_{x=\pm 1} = 0, \quad (c_y - \varepsilon \theta_y)|_{y=\pm \delta} = 0. \quad (12)$$

The continuity equation $u_x = -v_y$ allows us to introduce a stream-function $\psi(t, x, y)$:

$$u = \psi_y, \quad v = -\psi_x. \quad (13)$$

Taking into account (13), equation (5) and boundary conditions (9), we write

$$\begin{aligned} \frac{\partial}{\partial t} \Delta_2 \psi &= \Delta_2 \Delta_2 \psi + \theta_y + c_y, \\ \psi_{yy}(t, 1, y) &= \psi_{yy}(t, -1, y) = \psi_{yy}(t, x, \delta) = \psi_{yy}(t, x, -\delta) = 0, \\ \psi_{xx}(t, 1, y) &= \psi_{xx}(t, -1, y) = \psi_{xx}(t, x, \delta) = \psi_{xx}(t, x, -\delta) = 0. \end{aligned} \quad (14)$$

To complete formulation of the problem we use initial conditions and compatibility conditions. Then the stream-function satisfy the following conditions

$$\begin{aligned} \psi(0, x, y) &= \psi_0(x, y), \\ \psi_{0yy}(1, y) &= \psi_{0yy}(-1, y) = \psi_{0yy}(x, \delta) = \psi_{0yy}(x, -\delta) = 0, \\ \psi_{0xx}(1, y) &= \psi_{0xx}(-1, y) = \psi_{0xx}(x, \delta) = \psi_{0xx}(x, -\delta) = 0. \end{aligned} \quad (15)$$

Conditions for the third component of the velocity vector take the form

$$\begin{aligned} w(0, x, y) &= w_0(x, y), \\ w_0(1, y) &= w_0(-1, y) = w_0(x, \delta) = w_0(x, -\delta) = 0. \end{aligned} \quad (16)$$

For functions $\theta(t, x, y)$ and $c(t, x, y)$ we have

$$\begin{aligned} \theta(0, x, y) &= \theta_0(x, y), \\ \theta_0(1, y) &= \theta_1^1(y), \quad \theta_0(-1, y) = \theta_2^1(y), \quad \theta_0(x, \delta) = \theta_1^2(x), \quad \theta_0(x, -\delta) = \theta_2^2(y). \end{aligned} \quad (17)$$

$$\begin{aligned} c(0, x, y) &= c_0(x, y), \\ c_{0x}(1, y) - \varepsilon \theta_{0x}(1, y) &= 0, \quad c_{0x}(-1, y) - \varepsilon \theta_{0x}(-1, y) = 0, \\ c_{0y}(x, \delta) - \varepsilon \theta_{0y}(x, \delta) &= 0, \quad c_{0y}(x, -\delta) - \varepsilon \theta_{0y}(x, -\delta) = 0. \end{aligned} \quad (18)$$

2. The solution of non-stationary problem

Let us consider equation (6) with boundary condition (10) and initial condition (16).

We solve the first boundary value problem by the method of separation of variables [5]. We seek a solution in the form of a double series

$$w(t, x, y) = \sum_{n,k=0}^{\infty} X_k(x) Y_n(y) T_{kn}(t).$$

Function x is also represented in the form of a series

$$x = \sum_{n,k=0}^{\infty} X_k(x) Y_n(y) f_{kn}(t),$$

The coefficient $f_{kn}(t)$ is defined later. After substituting the series into equation (6) we obtain

$$\frac{T'_{nk} + f_{nk}}{T_{kn}} = \frac{X''_k}{X_k} + \frac{Y''_n}{Y_n}, \quad k, n \in N,$$

where

$$\frac{X''_k}{X_k} + \frac{Y''_n}{Y_n} = -\lambda_{kn}, \quad \mu_k + \nu_n = \lambda_{kn}.$$

The eigenvalues μ_k and ν_n can be found from Sturm-Liouville problem for $X_k(x)$ and $Y_n(y)$:

$$\begin{aligned} X_k(x) &= \sin(\pi k x), \quad \mu_k = (\pi k)^2, \\ Y_n(y) &= \sin\left(\frac{\pi n}{\delta} y\right), \quad \nu_n = \left(\frac{\pi n}{\delta}\right)^2. \end{aligned}$$

Now we need to solve the Cauchy problem

$$T'_{kn} + f_{kn} = -KT_{kn}, \quad T_{kn}(0) = K_{w_0},$$

where $K = \pi^2 \delta^{-2} (\delta^2 k^2 + n^2)$ and coefficient K_{w_0} follows from the Fourier sine series expansion for the function $w_0(x, y)$:

$$K_{w_0} = \frac{1}{\|w_0\|} \iint_{\Gamma} w_0(x, y) \sin(\pi k x) \sin\left(\frac{\pi n}{\delta} y\right) dx dy.$$

$$\text{Coefficients } \|w_0\| \text{ and } f_{nk} \text{ are } \|w_0\| = \left(\int_{-1}^1 \int_{-\delta}^{\delta} w_0^2 dx dy \right)^{1/2},$$

$$f_{nk} = \sqrt{3}(2\sqrt{\delta})^{-1} \int_{-1}^1 \int_{-\delta}^{\delta} x \sin(\pi k x) \sin\left(\frac{\pi n}{\delta} y\right) dx dy = 4\sqrt{3}\delta\pi^{-2}(nk)^{-1}((-1)^n - 1)(-1)^k.$$

Finally, the solution of problem (6), (10), (16) has the form

$$w(t, x, y) = \sum_{n,k=0}^{\infty} \left(\left(K_{w_0} + \frac{f_{nk}}{K} \right) \exp(-Kt) - \frac{f_{nk}}{K} \right) \sin(\pi k x) \sin\left(\frac{\pi n}{\delta} y\right).$$

Function $\theta(t, x, y)$ is the solution of the boundary value problem (7), (11), (17). The solution can be obtained with the use of the Green's function as [6]

$$\begin{aligned} \theta(t, x, y) &= \frac{1}{Pr} \int_0^t \int_0^2 \int_0^{2\delta} w(\tau, \xi, \eta) G(t - \tau, x + 1, y + \delta, \xi, \eta) d\xi d\eta d\tau + \\ &\quad + \int_0^2 \int_0^{2\delta} \theta_0(\xi, \eta) G(t, x + 1, y + \delta, \xi, \eta) d\eta d\xi + \\ &\quad + \frac{1}{Pr} \int_0^t \int_0^{2\delta} \theta_2^1(\eta, \tau) \left(\frac{\partial}{\partial \xi} G(t - \tau, x + 1, y + \delta, \xi, \eta) \right) \Big|_{\xi=0} d\eta d\tau - \\ &\quad - \frac{1}{Pr} \int_0^t \int_0^{2\delta} \theta_1^1(\eta, \tau) \left(\frac{\partial}{\partial \xi} G(t - \tau, x + 1, y + \delta, \xi, \eta) \right) \Big|_{\xi=2} d\eta d\tau + \\ &\quad + \frac{1}{Pr} \int_0^t \int_0^2 \theta_2^2(\xi, \tau) \left(\frac{\partial}{\partial \eta} G(t - \tau, x + 1, y + \delta, \xi, \eta) \right) \Big|_{\eta=0} d\xi d\tau - \\ &\quad - \frac{1}{Pr} \int_0^t \int_0^2 \theta_1^2(\xi, \tau) \left(\frac{\partial}{\partial \eta} G(t - \tau, x + 1, y + \delta, \xi, \eta) \right) \Big|_{\eta=2\delta} d\xi d\tau. \end{aligned}$$

The Green's function is written as

$$G(t, x, y, \xi, \eta) = \frac{1}{\delta} \sum_{k,n=0}^{\infty} F(x, y, \xi, \eta) \exp\left(-\frac{Kt}{4Pr}\right)$$

where $F(x, y, \xi, \eta) = \sin\left(\frac{1}{2}\pi kx\right) \sin\left(\frac{1}{2}\pi k\xi\right) \sin\left(\frac{1}{2\delta}\pi ny\right) \sin\left(\frac{1}{2\delta}\pi n\eta\right)$.

Let us consider the solution of boundary value problem (8), (12), (18). Because we have inhomogeneous boundary conditions for the function $c(t, x, y)$, we introduce new unknown function $c(t, x, y) = \tilde{c}(t, x, y) + \varepsilon\theta(t, x, y)$. Now we have boundary value problem with zero boundary conditions for $\tilde{c}(t, x, y)$:

$$\begin{aligned} S\tilde{c}_t &= \tilde{c}_{xx} + \tilde{c}_{yy} - S\varepsilon\theta_t + \varepsilon_1 w, \\ \tilde{c}_x|_{x=\pm 1} &= 0, \quad \tilde{c}_y|_{y=\pm\delta} = 0, \\ \tilde{c}(0, x, y) &= c_0(x, y) - \varepsilon\theta_0(x, y). \end{aligned} \tag{19}$$

Function $-S\varepsilon\theta_t + \varepsilon_1 w$ is known.

In order to solve problem (19) we divide it into two subproblems. The first problem is homogeneous problem with nonzero initial conditions. The second problem is inhomogeneous problem with zero initial conditions. The solution of these boundary value problems can be obtained by the method of separation of variables. Finally, the solution of problem (8), (12), (18) has the form:

$$c(t, x, y) = \sum_{k,n=0}^{\infty} \left(K_{c_0\theta_0} \exp\left(-\frac{Kt}{Sc}\right) + \frac{K_{w\theta}}{K} \left(1 - \exp\left(-\frac{Kt}{Sc}\right)\right) \right) \cos(\pi kx) \cos\left(\frac{\pi n}{\delta}y\right) + \varepsilon\theta,$$

where $K_{c_0\theta_0}$ are coefficients of Fourier cosine series expansion for initial condition (19), $K_{w\theta}$ are coefficients of Fourier cosine series expansion for the known function on the right-hand side of equation (19).

It only remains for us to solve boundary value problem (14), (15) to find stream-function. The stream-function is represented in the following form:

$$\psi(t, x, y) = \sum_{k,n=0}^{\infty} \psi_{kn}(t) \sin(\pi kx) \sin\left(\frac{\pi n}{\delta}y\right).$$

Let us notice that functions $\sin(\pi kx)$ and $\sin(\pi n\delta^{-1}y)$ are orthogonal on $-1 \leq x \leq 1$, $-\delta \leq y \leq \delta$, that is, $\int_{-1}^1 \int_{-\delta}^{\delta} \sin(\pi kx) \sin(\pi n\delta^{-1}y) dx dy = 0$.

After substituting the expression for the stream-function in equation (14) we obtain function $\psi_{kn}(t)$ and the solution takes the form:

$$\psi(t, x, y) = \sum_{k,n=0}^{\infty} \left(-\frac{K_{\theta c}}{K^2} + \left(\frac{K_{\theta c}}{K^2} + K_{\psi_0} \right) \exp(-Kt) \right) \sin(\pi kx) \sin\left(\frac{\pi n}{\delta}y\right),$$

where $K_{c\theta}$ are coefficients of Fourier sine series expansion for the known function on the right-hand side of equation (14), K_{ψ_0} are coefficients of Fourier sine series expansion for initial condition (15).

3. The solution of stationary problem

Now we consider stationary problem. One needs to solve the system of equations

$$\begin{aligned}\Delta_2 \Delta_2 \psi + \theta_y + c_y &= 0, \\ -x + \Delta_2 w &= 0, \\ -w &= \Delta_2 \theta, \\ -\varepsilon_1 w &= \Delta_2 c - \varepsilon \Delta_2 \theta.\end{aligned}\tag{20}$$

with boundary conditions (5)–(8), (10)–(12), (14)

$$\begin{aligned}\psi_{yy}(\pm 1, y) &= \psi_{yy}(x, \pm \delta) = 0, \\ \psi_{xx}(\pm 1, y) &= \psi_{xx}(x, \pm \delta) = 0, \\ w(\pm 1, y) &= w(x, \pm \delta) = 0, \\ \theta(\pm 1, y) &= \theta_1^{1,2}(y), \theta(x, \pm \delta) = \theta_2^{1,2}(x), \\ (c_x - \varepsilon \theta_x)|_{x=\pm 1} &= 0, (c_y - \varepsilon \theta_y)|_{y=\pm \delta} = 0.\end{aligned}\tag{21}$$

Problem (20), (21) is solved by the method of separation of variables with the use of Green's function [6]. The solution has the form

$$w(x, y) = - \sum_{k,n=0}^{\infty} \frac{f_{nk}}{K} \sin(\pi k x) \sin\left(\frac{\pi n}{\delta} y\right),$$

$$\theta(x, y) =$$

$$\begin{aligned}&= - \int_0^2 \int_0^{2\delta} w(\xi, \eta) G_1(x+1, y+\delta, \xi, \eta) d\xi d\eta + \int_0^{2\delta} \theta_2^1(\eta) \left(\frac{\partial}{\partial \xi} G_1(x+1, y+\delta, \xi, \eta) \right) \Big|_{\xi=0} d\eta - \\ &- \int_0^{2\delta} \theta_1^1(\eta) \left(\frac{\partial}{\partial \xi} G_1(x+1, y+\delta, \xi, \eta) \right) \Big|_{\xi=2} d\eta + \int_0^2 \theta_2^2(\xi) \left(\frac{\partial}{\partial \eta} G_1(x+1, y+\delta, \xi, \eta) \right) \Big|_{\eta=0} d\xi - \\ &- \int_0^2 \theta_1^2(\xi, \tau) \left(\frac{\partial}{\partial \eta} G_1(x+1, y+\delta, \xi, \eta) \right) \Big|_{\eta=2\delta} d\xi,\end{aligned}$$

$$c(x, y) = \varepsilon \theta + \sum_{k,n=0}^{\infty} \frac{K_w}{K} \cos(\pi k x) \cos\left(\frac{\pi n}{\delta} y\right),$$

$$\psi(x, y) = - \sum_{k,n=0}^{\infty} \frac{K_{\theta c}}{K^2} \sin(\pi k x) \sin\left(\frac{\pi n}{\delta} y\right),$$

where $G_1(x, y, \xi, \eta) = \frac{4}{\delta} \sum_{k,n=0}^{\infty} \frac{1}{K} F(x, y, \xi, \eta)$ and K_w is Fourier cosine transformation for the function $\varepsilon_1 w$.

4. Convergence of the non-stationary solution to the stationary solution

Let us consider convergence of the solution of non-stationary problem to the solution of stationary problem. Let us denote the solution of stationary problem as w_s, θ_s, c_s , and ψ_s .

It is easy to see that when $t \rightarrow \infty$ function $w(t, x, y)$ tends to function $w_s(x, y)$. The difference between $w(t, x, y)$ and $w_s(x, y)$ is

$$\begin{aligned} |w_s - w| &= \\ &= \left| \sum_{n,k=0}^{\infty} \left(\left(K_{\omega_0} + \frac{f_{nk}}{K} \right) \exp(-Kt) - \frac{f_{nk}}{K} \right) \sin(\pi kx) \sin\left(\frac{\pi n}{\delta}y\right) + \right. \\ &\quad \left. + \sum_{k,n=0}^{\infty} \frac{f_{nk}}{K} \sin(\pi kx) \sin\left(\frac{\pi n}{\delta}y\right) \right| \leq \\ &\leq \sum_{n,k=0}^{\infty} \left| \left(K_{\omega_0} + \frac{f_{nk}}{K} \right) \exp(-Kt) \right| \xrightarrow{t \rightarrow \infty} 0. \end{aligned}$$

Let us consider functions $c(t, x, y)$ and $c_s(x, y)$. The difference between these functions depend on the difference between $S\varepsilon\theta_t + \varepsilon_1 w$ and $\varepsilon_1 w$. For stationary problem $\theta_t = 0$, that is, $(Sc\varepsilon\theta_t + \varepsilon_1 w)|_{\theta_t=0} = \varepsilon_1 w$. Then we have

$$|c_s - c| \leq \sum_{n,k=0}^{\infty} \left| \frac{1}{K} (K_{\omega\theta} - K_{\omega}) \right| \rightarrow 0.$$

Therefore, the solution of non-stationary problem $c(t, x, y)$ converges to the solution of stationary problem.

In a similar way, we can show the convergence of function $\psi(t, x, y)$ to function $\psi(x, y)$ when $t \rightarrow \infty$.

$$|\psi_s - \psi| \leq \sum_{n,k=0}^{\infty} \left| \frac{K_{\theta c}}{K^2} - \frac{K_{\theta c}}{K^2} + \left(\frac{K_{\theta c}}{K^2} + K_{\psi_0} \right) \exp(-Kt) \right| \xrightarrow{t \rightarrow \infty} 0.$$

Now we prove convergence of function $\theta(t, x, y)$ to function $\theta_s(x, y)$ with the use of the maximum principle [5]. If $T = \theta - \theta_s$ then we obtain from equation (7) and the third equation (20) the following equation

$$T_t = \frac{1}{Pr} \Delta_2 T + \frac{1}{Pr} (\omega - \omega_s).$$

Boundary conditions for this equation follows from (11) and (21):

$$T(t, \pm 1, y) = \theta_{1,2}^1(t, y) - \theta_{1,2}^1(y), \quad T(t, x, \pm \delta) = \theta_{1,2}^2(t, x) - \theta_{1,2}^2(x),$$

Initial condition is $T(0, x, y) = \theta_0(x, y)$ and we should also add the compatibility conditions. According to the maximum principle, the maximum value of the function $T(t, x, y)$ is achieved on the boundary:

$$|T(t, x, y)| \leq \max_{t \in [0, T]} (T(t, 1, y); T(t, -1, y); T(t, x, \delta); T(t, x, -\delta); \omega - \omega_s; \theta_0).$$

If $t \rightarrow \infty$ then $T(t, \pm 1, y) \rightarrow 0$, $T(t, x, \pm \delta) \rightarrow 0$, as $\theta_j^1(t, y) \rightarrow \theta_j^1(y)$, $\theta_j^2(t, x) \rightarrow \theta_j^2(x)$, $j = 1, 2$. It was proved earlier that $\omega - \omega_s \rightarrow 0$. It follows from compatibility condition that $\theta_0 \rightarrow 0$. Therefore, $T(t, x, y) \rightarrow 0$ and we proved that $|\theta - \theta_s|_{\overline{t \rightarrow \infty}} \rightarrow 0$.

Similar result has been obtained in the case of creeping motion in a horizontal cylindrical tube [1].

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О ползущем движении бинарной смеси в трубе прямоугольного сечения

Александра Е. Силаева

В работе дана постановка задачи движения бинарной смеси в трубе прямоугольного сечения. Получены точные решения стационарной и нестационарной задач в виде рядов. Доказано, что с ростом времени решение выходит на стационарный режим.

Ключевые слова: бинарная смесь, стационарная задача, нестационарная задача, ползущие решения.