# Research of accuracy characteristics of measurement of coordinates in the ground-based radionavigation system based on pseudosatellites

A B Gladyshev<sup>1</sup>, D D Dmitriev<sup>1</sup>, V N Ratuschnyak<sup>1</sup>, M A Golubyatnikov<sup>2</sup>, I N Kartsan<sup>3</sup> and R Yu Tsarev<sup>1</sup>

E-mail: kartsan2003@mail.ru

**Abstract**. The issues of measurement accuracy of coordinates in the short-range navigation system based on pseudosatellites are considered in the article. The analysis of the errors components the of measurement of radio-navigation parameters is given, the values of the geometrical factor are calculated for various options of system constructing.

### 1. Introduction

High-precision positioning technologies are being implemented more actively using radio navigation systems in transport control systems. The reason for the introduction is the increasing density of traffic flows, especially in air and on water.

However, despite the great popularity of global navigation satellite systems (GNSS), they are rarely used for autonomous navigation support of such activities as the provision of aviation flights in aerodrome zones; flight support of unmanned aircraft; navigation in port areas, narrow areas, on rivers; providing geodetic, cartographic and special works.

The reason for such a cautious attitude to GNSS is the low noise immunity. Due to this it is not always possible to fulfill the requirements of the continuity and accuracy of the navigation support of this activity.

One of the ways to resolve this contradiction is the addition of GNSS ground-based radio navigation systems of short range navigation, which allows you to create a navigation field in a given region of space using ground-based pseudosatellites (PS).

High-precision geodetic coupling and mutual synchronization of PSs ensure high accuracy of the consumer's measurement of the navigation parameters of such a radio navigation systems. PS is a stationary emitters of navigation signals, which are similar to signals of navigation spacecraft GLONASS.

The similar architecture of the short-range navigation system is significantly different from the local, regional or wide-gap differential GNSS subsystems. If the use of GNSS subsystems is possible only with a stable reception of GLONASS signals, then using the navigation system based on PS it is possible to provide navigation even in the absence of signals from the navigation spacecraft.

<sup>&</sup>lt;sup>1</sup>Siberian Federal University, Krasnovarsk, Russia

<sup>&</sup>lt;sup>2</sup>Student, Siberian Federal University, Krasnoyarsk, Russia

<sup>&</sup>lt;sup>3</sup>Reshetnev Siberian State University of Science and Technology, Krasnoyarsk, Russia

# 2. Errors of measurement of coordinates in the ground-based radionavigation system

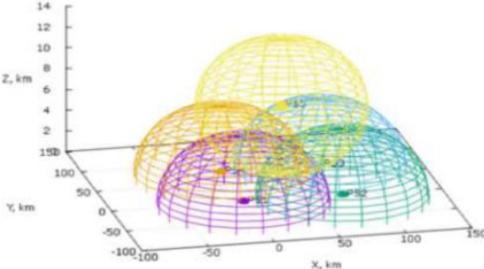
The location of pseudo-satellites on the ground, their number in the short-range-navigation system under consideration depend on the terrain relief and the size of the area where this system should be used. In areas of airports, this area of space has to include the glide slope for landing aircraft, and it has to include the fairway on rivers. This area is covered by a zone where high-precision continuous navigation should be ensured, regardless of the prevailing meteorological conditions, jamming environment, etc.

Various factors influence the accuracy of measuring the coordinates of a moving object in a given region of space. We give only the main ones:

- the geometry of the location of the PS relative to the navigation receiver (the so-called geometric factor);
- pseudorange measurement error due to the structure of navigation signals and the method of processing;
- error of geodesic referencing of PS and of the quality of their mutual synchronization with each other and with the scale of the national standard of coordinated universal time UTC (SU) (analogous to the ephemeris error in GNSS);
  - the errors introduced by the medium, the channel and multipath signal propagation;
  - the ratio of energy in the resulting mixture of the useful signal and interference.

The complex task of choosing the optimal location of the PS on the ground, the power of the emitted navigation signals, the structure of tracking schemes of navigation receivers, which will ensure the specified measurement accuracy and navigation area (figure 1) arises during the development on the short-range-navigation system based on PS. The error in measuring the coordinates of an object will depend on the hardware error in measuring the pseudorange Ri, the error in measuring the coordinates of the PS and the errors introduced by the signal propagation channel.

The value of the hardware error of the pseudorange measurement will be approximately equal to the analogous error in GNSS and will be about 1.5 m for the ground-based short-range navigation system based on PS, using signals similar to GNSS signals [1].



**Figure 1.** Radio navigation field of five PS.

The error in ground-based short-range navigation systems arising due to inaccurate determination of PS coordinates and their mutual synchronization (an analog of ephemeris error in GNSS) will be significantly less than in GNSS.

This is due to the highly accurate geodesic referencing of PS and the possibility of their mutual synchronization using highly stable time and frequency standards.

The effect of multipath error can be reduced by averaging the measurement results over time intervals (the multipath error correlation interval) and compiling a multipath map for terrain features.

In the system under consideration, we can disregard the error caused by the channel and the medium of signal propagation due to sufficiently small distances [2].

The possibility of planning and external error due to the geometrical factor is one of the advantages of the system under consideration, unlike GNSS.

The location of the PS on the ground can be chosen in such a way as to minimize the impact of its on the responsible section of navigation.

The essence of the geometric factor is as follows. The system of nonlinear equations (1), the number of which is determined by the number of PS's, is used to calculate the coordinates of the object.

$$R_{i} = \sqrt{(X_{mi} - X)^{2} + (Y_{mi} - Y)^{2} + (Z_{mi} - Z)^{2}} + c\Delta t,$$
(1)

where  $X_{mi}$ ,  $Y_{mi}$ ,  $Z_{mi}$  are the coordinates of the *i*-th PS; X, Y, Z – coordinates of the navigation receiver; c – is the speed of light;  $\Delta t$  – systematic error caused by the discrepancy of time scales.

The nonlinearity of the equations is characterized by the radius of curvature of the wave front, which is equal to the distance from the object to the PS.

We can linearize the system of equations at the point of reception of signals. Since the radius of curvature of the wave front is much larger than the measurement error of the range, the error conversion will be close to linear.

In the case of a linear transformation, the error in determining the coordinates is described by a covariance matrix. The covariance matrix can be obtained from the gradient matrix:

$$cov(r) = Gr^{T}Gr, (2)$$

where *Gr* is a gradient matrix:

$$Gr = \begin{vmatrix} \frac{\partial R_1}{\partial X} & \frac{\partial R_1}{\partial Y} & \frac{\partial R_1}{\partial Z} & \frac{\partial R_1}{\partial (C\Delta t)} \\ \frac{\partial R_2}{\partial X} & \frac{\partial R_2}{\partial Y} & \frac{\partial R_2}{\partial Z} & \frac{\partial R_2}{\partial (C\Delta t)} \\ \dots & \dots & \dots & \dots \\ \frac{\partial R_N}{\partial X} & \frac{\partial R_N}{\partial Y} & \frac{\partial R_N}{\partial Z} & \frac{\partial R_N}{\partial (C\Delta t)} \end{vmatrix} = \begin{vmatrix} \frac{X_{m1} - X}{R_1} & \frac{Y_{m1} - Y}{R_1} & \frac{Z_{m1} - Z}{R_1} & 1 \\ \frac{X_{m2} - X}{R_2} & \frac{Y_{m2} - Y}{R_2} & \frac{Z_{m2} - Z}{R_2} & 1 \\ \dots & \dots & \dots & \dots \\ \frac{X_{mN} - X}{R_N} & \frac{Y_{mN} - Y}{R_N} & \frac{Z_{mN} - Z}{R_N} & 1 \end{vmatrix}, \quad (3)$$

where  $(X_{mN} - X) / R_{iN} = \cos\alpha_i$ ;  $(Y_{mN} - Y) / R_{iN} = \cos\beta_i$ ;  $(Z_{mN} - Z) / R_{iN} = \cos\gamma_i$  – direction cosines of radius vectors connecting the *N*-th consumer and the *i*-th PS;  $R_{iN}$  – range between the *i*-th PS and the *N*-th consumer.

We can separate measurement errors and transformations if the pseudoranges measurements are equal. In this case, the error in determining the coordinates will be equal to the product of the covariance matrix and the variance of the distance measurement. We estimate the resulting error through the trace of the covariance matrix.

In the general case, the covariance matrix may contain nonzero elements outside the main diagonal, which describe the correlation of the errors of the individual components of the coordinates.

The elements of the main diagonal are the variances of the coordinates components; the resulting variance will be equal to their sum.

At the same time, the sum of the diagonal elements of the covariance matrix does not change under a linear transformation, since it is a linear invariant of the rank 2 tensor.

Therefore, the trace of the matrix is a coefficient that shows how many times the dispersion of coordinates increases as compared with the dispersion of the measured distance. The coefficients of the gradient matrix are directional cosines on the PS.

In practice, the square root of the covariance matrix trace is used, which shows the degree of increase in the root-mean-square (RMS) error in determining the coordinates compared to the RMS of the measurement error of the distance:

$$\sigma_{x,y} = \sqrt{\operatorname{cov}(r_{11}) + \operatorname{cov}(r_{22})}, \quad \sigma_z = \sqrt{\operatorname{cov}(r_{33})},$$
(4)

where  $\sigma_{x,y}$ ,  $\sigma_z - RMS$  are the errors in determining the coordinates in the plan and in height, respectively [3].

We calculate the values of the geometric factor for the short-range navigation zone using the example of the Yemelyanovo International Airport in Krasnoyarsk using mathematical modeling in the Mathlab software environment.

The required size of the navigation zone in the landing zone of Yemelyanovo Airport, which meets the requirements of ICAO, is shown in figure 2, where the number 1 indicates the required zone of glide navigation (in height and width is 1 km, and 27.79 km in length).



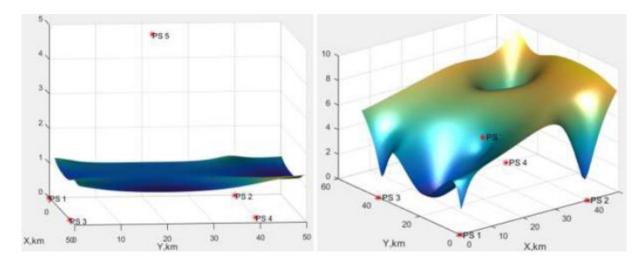
Figure 2. Airport glide zone.

For ground-based radio navigation systems with PS located at the equal height from the Earth surface, it is almost impossible to minimize the height measurement error due to a large geometric factor that does not meet the performance requirements in accordance with ICAO standards. The geometric factor of the height measurement of coordinates is significantly larger than the geometric factor of the measurement of the planned coordinates. This effect is explained by the fact that PS are located in one plane, and when an object approaches this plane, the height measurement error increases, while the error in the planned coordinates is almost independent on the height of the object and is 1 ... 2 [3, 4].

The first option for resolving this contradiction can be placing one or several PS on high-rise mast devices, lifting devices (balloons, etc.) or using the terrain characteristics at nearby dominant heights. From an economic and technical point of view, the task of locating PSs at these heights is laborious, and in some cases its solution is not advisable.

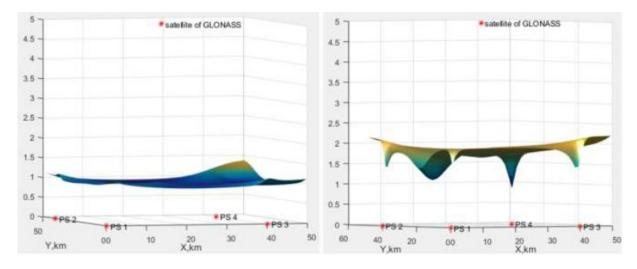
In the course of the simulation, the geometric factor of the horizontal and vertical planned coordinates was determined using the configuration of five PS. Figure 3a, 3b show the calculated values of the geometric factor for measurement of the planned coordinates and of the height, respectively.

The second solution to the task of reducing the geometric factor on the vertical coordinates is the joint reception of signals from ground-based PS and navigation spacecrafts of GLONASS by consumers of navigation information. In the case of air vehicles, the receiving equipment must provide the reception of GNSS signals from the upper hemisphere, and PS signals from the lower one, and process the received signals in a single radio path to reduce the systematic error of the receiving equipment and hardware costs [5].



**Figure 3.** The values of the geometric factor: a - in plane; b - on height.

To calculate the geometric factor in the joint reception of PS and navigation spacecrafts signals, the program uses the almanac and ephemeris data of the satellite system to calculate the spatial attitude of the GLONASS satellite constellation. When choosing an navigation spacecraft in the zenith, the value of the geometric factor on vertical coordinates is not more than 2 (figure 4).



**Figure 4.** The values of the geometric factor: a - in plane; b - on height.

In this configuration of the usage of PS and navigation spacecrafts signals the range of values of the geometric factor on vertical coordinates meets the requirements of the first category of ICAO.

Thus, the lifting to the altitude of one or more PS or the method of joint reception of navigation spacecrafts GNSS signals with the signals of ground-based PS provides the required accuracy of the measurement of vertical coordinates for air, ground and water kinds of transport.

### 3. Conclusion

Thus, the ability to select different PS locations on the ground allows the navigation field to be adapted for a selected object navigation zone, taking into account the terrain features, and also to form the required geometric factor in a given area of space. It is desirable to use 5...10 PS, placed evenly around the perimeter of the service area and several PS, placed at different heights to improve the

quality of navigation support in height. For a more accurate measurement of the height of aircraft, it is recommended to combine stand-alone on-board barometers, radio altimeters and GNSS receivers with receivers of a short-range navigation system based on PS.

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