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Determining Hydraulic Friction Factor for Pipeline Systems

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A comparative analysis of many well-known formulas for Darcy friction factor was carried out to determine accuracy and computational costs. To ensure a smooth transition from laminar flow to turbulent a cubic interpolation algorithm proposed to cover critical zone.

Keywords: hydraulic friction factor, critical zone, Darcy friction factor, pipeline systems, interpolation.

Introduction

The core of all known methods of analyzing the hydrodynamic state in regulated pipeline systems are methods of calculating flow distribution [1], [2], and all of them require calculation of hydraulic friction factor λ, which depends on the surface of the pipe wall, and the flow mode of the liquid. Determination of λ in the critical zone between laminar and transitional flows (Fig. 1) is related to certain difficulties. The goal of this article is to systematize the known methods of calculating λ and offer readers a general approach to the definition of λ on the whole range of Reynolds numbers.

Models and algorithms used

Head loss in a steady flow of liquid in round pressure pipes is calculated using Darcy-Weisbach equation

$$\Delta H = \left(\lambda \frac{l}{d_{int}} + \sum \xi\right) \frac{v^2}{2\,g}, \tag{1}$$

where l, d_{int} – length and inner pipe diameter, m; Σξ – sum of minor loss coefficients; v – velocity of fluid, m/s; g – gravitational acceleration, m/s^2.

Equation (1) obviously shows importance of valid definition of friction factor, which has at least the same impact weight as length of a pipe. When Σξ = 0 deviations of both λ and l have linear impact on total headloss.

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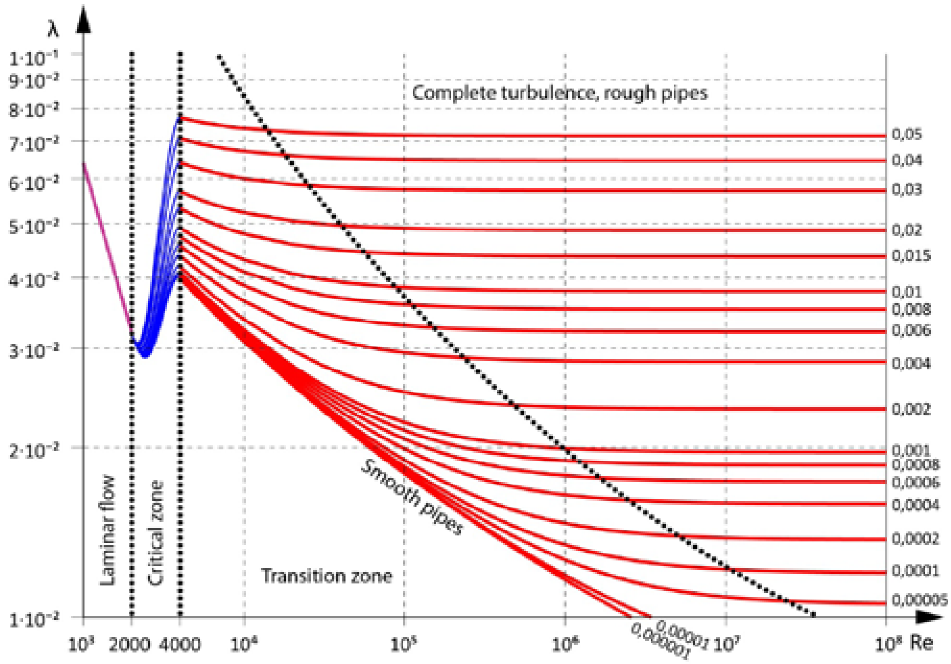

Fig. 1. Classical Moody chart for friction factor as function of *k_e/d_int* and *Re* reproduced with proposed model for critical zone

For laminar flow, with small Reynolds number $Re < 2300$, headloss depends on physical properties of fluid (viscosity and density) and its velocity, and does not depend on pipe inner walls roughness height, hydraulic friction factor is given by Poiseuille equation (1840)

$$\lambda = 64/\mathrm{Re}. \tag{2}$$

For turbulent flow in smooth pipes (the roughness of inner tube surface covered with laminar sublayer) Blasius (1913) equation can be used, which is valid for $4000 \leq Re \leq 100000$

$$\lambda = 0{,}3164/\mathrm{Re}^{0{,}25}. \tag{3}$$

For hydraulically smooth pipes Prandtl (1932) proposed formula

$$\frac{1}{\sqrt{\lambda}} = 2\, lg\left(\mathrm{Re}\,\sqrt{\lambda}\right) - 0{,}8. \tag{4}$$

For hydraulically smooth pipes also known Altshul equation ($Re \geq 10^4$)

$$\lambda = 1/(1{,}82\ \mathrm{lgRe} - 1{,}64)^2 \tag{5}$$

and Nikuradse equation ($Re \geq 10^5$)

$$\lambda = 0{,}0032 + 0{,}221/\mathrm{Re}^{0{,}237}. \tag{6}$$

Colebrook-White (1939) equation describes behaviour of hydraulic friction factor with $Re > 4000$ in conduits that are flowing completely full of fluid for smooth and rough pipes.

$$\frac{1}{\sqrt{\lambda}} = -2\ \log_{10}\left(\frac{k_e}{3,7\ d_{int}} + \frac{2,51}{\mathrm{Re}\ \sqrt{\lambda}}\right), \quad (7)$$

where $k_в$ – roughness height of inner tube surface, m.

Because of implicit nature of Colebrook equation (7) λ is obtained either numerically, or by composing approximation formulas. Recently, the Lambert W function was used to get explicit form of (7).

For transition zone of turbulent flow between smooth and rough pipes Altshul equation can be used in hydraulic calculations of thermal pipeline networks

$$\lambda = 0,11\left(\frac{k_e}{d_{int}} + \frac{68,5}{\mathrm{Re}}\right)^{0,25}, \quad (8)$$

For turbulent zone in the area of quadratic law of flow Prandtl-Nikuradse formula can be used

$$\lambda = \frac{1}{\left(1,14 + 2\lg\frac{d_{int}}{k_e}\right)^2} \quad (9)$$

and Shifrinson formula

$$\lambda = 0,11\left(\frac{k_e}{d_{int}}\right)^{0,25}. \quad (10)$$

Some of the other most known equations for friction factor are:

– Moody equation (1947)

$$\lambda = 0,0055\left(1 + \left(2 \cdot 10^4\frac{k_e}{d_{int}} + \frac{10^6}{\mathrm{Re}}\right)^{1/3}\right); \quad (11)$$

– Wood equation (1966)

$$\lambda = 0,094\left(\frac{k_e}{d_{int}}\right)^{0,225} + 0,53\left(\frac{k_e}{d_{int}}\right) + 88\left(\frac{k_e}{d_{int}}\right)^{0,44}\mathrm{Re}^{-\psi}, \quad (12)$$

where

$$\psi = 1,62\left(\frac{k_e}{d_{int}}\right)^{0,134}; \quad (13)$$

– Eck equation (1973)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left(\frac{k_e}{3,715 d_{int}} + \frac{15}{\mathrm{Re}}\right); \quad (14)$$

– Churchill equation (1973)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left(\frac{k_e}{3,71 d_{int}} + \left(\frac{7}{\mathrm{Re}}\right)^{0,9}\right); \quad (15)$$

– Jain and Swamee equation (1976)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left(\frac{k_e}{3,7 d_{int}} + \frac{5,74}{\mathrm{Re}^{0,9}}\right); \quad (16)$$

– Jain equation (1976)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left(\frac{k_e}{3{,}715d_{int}} + \left(\frac{6{,}943}{\text{Re}}\right)^{0{,}9}\right); \tag{17}$$

– another Churchill equation (1977)

$$\lambda = 8\left[\left(\frac{8}{\text{Re}}\right)^{12} + \frac{1}{(\Theta_1 + \Theta_2)^{1{,}5}}\right]^{\frac{1}{12}}, \tag{18}$$

where

$$\Theta_1 = \left[-2{,}457\ln\left[\left(\frac{7}{\text{Re}}\right)^{0{,}9} + 0{,}27\frac{k_e}{d_{int}}\right]\right]^{16}, \tag{19}$$

$$\Theta_2 = \left(\frac{37530}{\text{Re}}\right)^{16}, \tag{20}$$

– Chen equation (1979)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{k_e}{3{,}7065d_{int}} - \frac{5{,}0452}{\text{Re}}\lg\left(\frac{1}{2{,}8257}\left(\frac{k_e}{d_{int}}\right)^{1{,}1098} + \frac{5{,}8506}{\text{Re}^{0{,}8981}}\right)\right]; \tag{21}$$

– Round equation (1980)

$$\frac{1}{\sqrt{\lambda}} = 1{,}8\lg\left[\frac{\text{Re}}{0{,}135\,\text{Re}\left(\frac{k_e}{d_{int}}\right) + 6{,}5}\right]; \tag{22}$$

– Barr equation (1981)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{k_e}{3{,}7\,d_{int}} + \frac{5{,}158\lg\frac{\text{Re}}{7}}{\text{Re}\left(1 + \frac{\text{Re}^{0{,}52}}{29}\left(\frac{k_e}{d_{int}}\right)^{0{,}7}\right)}\right]; \tag{23}$$

– Zigrang and Sylvester equation (1982)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{k_e}{3{,}7\,d_{int}} - \frac{5{,}02}{\text{Re}}\lg\left(\frac{k_e}{3{,}7\,d_{int}} - \frac{5{,}02}{\text{Re}}\lg\left(\frac{k_e}{3{,}7\,d_{int}} + \frac{13}{\text{Re}}\right)\right)\right]; \tag{24}$$

or

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{k_e}{3{,}7\,d_{int}} - \frac{5{,}02}{\text{Re}}\lg\left(\frac{k_e}{3{,}7\,d_{int}} + \frac{13}{\text{Re}}\right)\right]; \tag{25}$$

– Haaland equation (1983)

$$\frac{1}{\sqrt{\lambda}} = -1{,}8\lg\left[\left(\frac{k_e}{3{,}7\,d_{int}}\right)^{1{,}11} + \frac{69}{\text{Re}}\right]; \tag{26}$$

– Serghides equation (1984)

$$\lambda = \left[\psi_1 - \frac{(\psi_2 - \psi_1)^2}{\psi_3 - 2\psi_2 + \psi_1}\right]^{-2} \tag{27}$$

or

$$\lambda = \left[4{,}781 - \frac{\left(\psi_1 - 4{,}781\right)^2}{\psi_2 - 2\psi_1 + 4{,}781}\right]^{-2}, \tag{28}$$

where

$$\psi_1 = -2\lg\left(\frac{k_e}{3{,}7\, d_{int}} + \frac{12}{\mathrm{Re}}\right), \tag{29}$$

$$\psi_2 = -2\lg\left(\frac{k_e}{3{,}7\, d_{int}} + \frac{2{,}51\,\psi_1}{\mathrm{Re}}\right), \tag{30}$$

$$\psi_3 = -2\lg\left(\frac{k_e}{3{,}7\, d_{int}} + \frac{2{,}51\,\psi_2}{\mathrm{Re}}\right), \tag{31}$$

– Manadilli equation (1997)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left(\frac{k_e}{3{,}7\, d_{int}} + \frac{95}{\mathrm{Re}^{0{,}983}} - \frac{96{,}82}{\mathrm{Re}}\right); \tag{32}$$

– Monzon, Romeo and Royo equation (2002)

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left\{\frac{k_e}{3{,}7065\, d_{int}} - \frac{5{,}0272}{\mathrm{Re}}\lg\left[\frac{k_e}{3{,}827\, d_{int}} \frac{4{,}657}{\mathrm{Re}}\lg\left(\left(\frac{k_e}{7{,}7918\, d_{int}}\right)^{0{,}9924} + \left(\frac{5{,}3326}{208{,}815 + \mathrm{Re}}\right)^{0{,}9345}\right)\right]\right\}; \tag{33}$$

– Dobromyslov equation (2004) [7]

$$\sqrt{\lambda} = 0{,}5\,\frac{\frac{b}{2} + \frac{\left(1{,}312\,(2-b)\,\lg\left(\frac{3{,}7\, d_{int}}{k_e}\right)\right)}{\lg(Re) - 1}}{\lg\left(\frac{3{,}7\, d_{int}}{k_e}\right)}, \tag{34}$$

where

$$b = 1 + \frac{\lg(Re)}{\lg(Re_{kv})}, \tag{35}$$

при $b > 2$, $b = 2$

$$Re_{kv} = 500 \cdot \frac{d_{int}}{k_e}, \tag{36}$$

– Goudar and Sonnad equation (2006)

$$\frac{1}{\sqrt{\lambda}} = 0{,}8686\ln\left[\frac{0{,}4587\,\mathrm{Re}}{(S - 0{,}31)^{\frac{S}{(S+1)}}}\right]; \tag{37}$$

where

$$S = 0{,}124\,\mathrm{Re}\,\frac{k_e}{d_{int}} + \ln(0{,}4587\,\mathrm{Re}), \tag{38}$$

– Rao and Kumar equation (2006) [6]

$$\frac{1}{\sqrt{\lambda}} = 2\lg\left(\frac{d_{int}}{2 \cdot k_e \cdot B}\right), \tag{39}$$

where

$$B = \left(\frac{a + b \cdot Re}{Re}\right) \cdot f(Re), \tag{40}$$

$$f(Re) = 1 - 0{,}55e^{-0{,}33\left[\ln\left(\frac{Re}{6{,}5}\right)\right]^2}, \tag{41}$$

$$a = 0{,}444, \tag{42}$$

$$b = 0{,}135, \tag{43}$$

– Vatankhah and Kouchakzadeh equation (2008)

$$\frac{1}{\sqrt{\lambda}} = 0{,}8686\ln\left[\frac{0{,}4587\,\mathrm{Re}}{(S - 0{,}31)^{\frac{S}{(S+0{,}9633)}}}\right], \tag{44}$$

where

$$S = 0{,}124\,\mathrm{Re}\,\frac{k_e}{d_{int}} + \ln(0{,}4587\,\mathrm{Re}); \tag{45}$$

– Buzzelli equation (2008)

$$\frac{1}{\sqrt{\lambda}} = \alpha - \left[\frac{\alpha + 2\lg\left(\frac{\mathrm{B}}{\mathrm{Re}}\right)}{1 + \frac{2{,}18}{\mathrm{B}}}\right], \tag{46}$$

where

$$\alpha = \frac{(0{,}744\ln(\mathrm{Re})) - 1{,}41}{\left(1 + 1{,}32\sqrt{\frac{k_e}{d_{int}}}\right)}, \tag{47}$$

$$\mathrm{B} = \frac{k_e}{3{,}7\,d_{int}}\mathrm{Re} + 2{,}51\,\alpha; \tag{48}$$

– Goudar and Sonnad approximation (2008) [4]

$$\frac{1}{\sqrt{\lambda}} = a\left[\ln\left(\frac{d}{q}\right) + D_{CFA}\right], \tag{49}$$

where

$$D_{CFA} = D_{LA}\left(1 + \frac{\frac{z}{2}}{(g+1)^2 + \left(\frac{z}{3}\right)(2g - 1)}\right), \tag{50}$$

$$D_{LA} = z \cdot \frac{g}{g+1}, \tag{51}$$

$$z = \ln\left(\frac{q}{g}\right), \tag{52}$$

$$g = bd + \ln\left(\frac{d}{q}\right), \tag{53}$$

$$q = s^{\frac{s}{s+1}}, \tag{54}$$

$$s = bd + \ln(d), \tag{55}$$

$$d = \frac{\ln(10)\,Re}{5{,}02}, \tag{56}$$

$$b = \frac{k_e}{3{,}7 \cdot d_{int}}, \tag{57}$$

$$a = \frac{2}{\ln(10)}, \tag{58}$$

– Avci and Kargoz equation (2009)

$$\lambda = \frac{6{,}4}{\left\{\ln(\mathrm{Re}) - \ln\left[1 + 0{,}01\ \mathrm{Re}\ \frac{k_e}{d_{int}}\left(1 + 10\sqrt{\frac{k_e}{d_{int}}}\right)\right]\right\}^{2{,}4}}; \tag{59}$$

– Evangleids, Papaevangelou and Tzimopoulos equation (2010)

$$\lambda = \frac{0{,}2479 - 0{,}0000947\ (7 - \lg \mathrm{Re})^4}{\left[\lg\left(\frac{k_e}{3{,}615\ d_{int}} + \frac{7{,}366}{\mathrm{Re}^{0{,}9142}}\right)\right]^2}; \tag{60}$$

– Brkić solution based on Lambert W-function (2011) [5]

$$\frac{1}{\sqrt{\lambda}} = -2\ \lg\left(\frac{k_e}{3{,}71\ d_{int}} + \frac{2{,}18\ S}{Re}\right), \tag{61}$$

where

$$S = \ln\left(\frac{Re}{1{,}816\ \ln\left(\frac{1{,}1\ Re}{\ln(1 + 1{,}1\ Re)}\right)}\right). \tag{62}$$

Didier Clamond [3] proposed (2009) a special algorithm of iterative calculation of λ, which gives accuracy close to limits of computer type *double* after two iterations. It requires calculation of logarithm once for initial estimation and one time per iteration.

$$\lambda = F^2, \tag{63}$$

where

$$K = \frac{k_e}{d_{int}}; \tag{64}$$

$$X1 = K\ Re\ 0{,}123968186335417556; \tag{65}$$

$$X2 = \ln(Re) - 0{,}779397488455682028; \tag{66}$$

$$F = X2 - 0{,}2; \tag{67}$$

$$repeat\ 2\ times \left\langle \begin{array}{l} E = \dfrac{\ln(X1 + F) + F - X2}{1 + X1 + F} \quad (68) \\ F = F - \dfrac{(1 + X1 + F + 0{,}5\ E)\ E\ (X1 + F)}{1 + X1 + F + E\ \left(1 + \frac{E}{3}\right)}; \quad (69) \end{array} \right.$$

$$F = 1{,}151292546497022842\,F. \tag{70}$$

Hydraulic regime in critical zone is neither laminar, nor turbulent. It is complex and unstable, and thus, there are no formulas to describe friction factor for this zone. It is often suggested to exclude calculations in this area. However, sustainable mathematical model requires smooth and continuous functions. To solve this problem we can construct interpolation curve between two regimes – laminar and turbulent. Dunlop cubic interpolation for $2000 \le \mathrm{Re} \le 4000$ is widely adopted, with its coefficients set to match boundary equations of Poiseiulle for laminar flow and Swamee-and-Jain for turbulent.

$$\lambda = (\mathrm{X1} + \mathrm{R}\,(\mathrm{X2} + \mathrm{R}\,(\mathrm{X3} + \mathrm{X4})\,)\,), \tag{71}$$

where

$$\mathrm{Y3} = -0{,}86859 \ln\left(\frac{\mathrm{k_e}}{3{,}7\ \mathrm{d_{int}}} + \frac{5{,}74}{4000^{0{,}9}}\right); \tag{72}$$

$$\mathrm{Y2} = \frac{\mathrm{k_e}}{3{,}7\ \mathrm{d_{int}}} + \frac{5{,}74}{\mathrm{Re}^{0{,}9}}; \tag{73}$$

$$\mathrm{FA} = \mathrm{Y3}^{-2}; \tag{74}$$

$$\mathrm{FB} = \mathrm{FA}\ \left(2 - \frac{0{,}00514215}{\mathrm{Y2\ Y3}}\right); \tag{75}$$

$$\mathrm{R} = \frac{\mathrm{Re}}{2000}; \tag{76}$$

$$\mathrm{X1} = 7\ \mathrm{FA} - \mathrm{FB}; \tag{77}$$

$$\mathrm{X2} = 0{,}128 - 17\ \mathrm{FA} + 2{,}5\ \mathrm{FB}; \tag{78}$$

$$\mathrm{X3} = -0{,}128 + 13\ \mathrm{FA} - 2\ \mathrm{FB}; \tag{79}$$

$$\mathrm{X4}\ =\ \mathrm{R}\ (0{,}032 - 3\ \mathrm{FA} + 0{,}5\ \mathrm{FB}). \tag{80}$$

Goal setting

Instability of hydraulic regime in critical zone does not allow analytical definition of friction factor, which is why it is often suggested to exclude this regime from calculations. But, if we build mathematical software to calculate flow distribution in complex pipeline networks, we prefer smooth and continuous functions.

Main goal of mathematical modeling of λ in critical zone is building interpolation curve between laminar flow and transition zone of turbulent flow.

Technique of calculation of hydraulic friction factor

A comparative analysis of existing formulas for Darcy friction factor for turbulent regime was carried out. Value of hydraulic friction factor, calculated by known formulas was substituted to original Colebrook-White equation (7), and absolute mean square deviation is shown on series of plots in fig. 2-6. Results show that lowest deviation (highest accuracy) gives Clamond method.

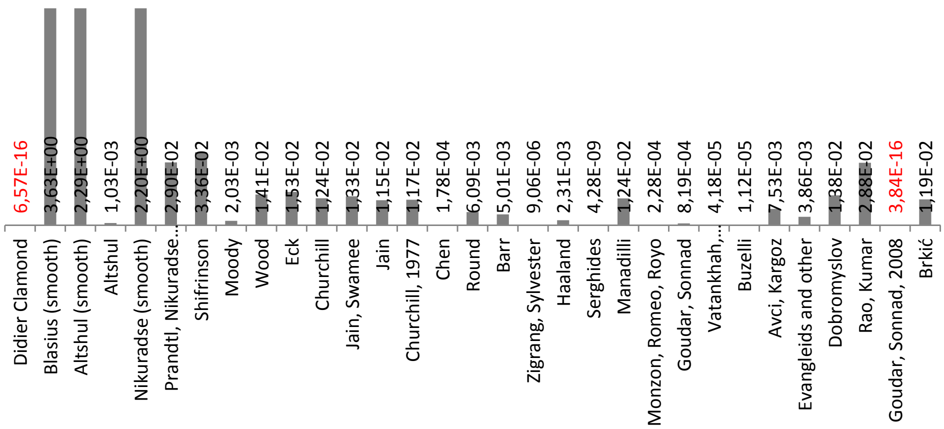

Fig. 2. Absolute mean square deviation for *k_e* / *d_int* = 0,05

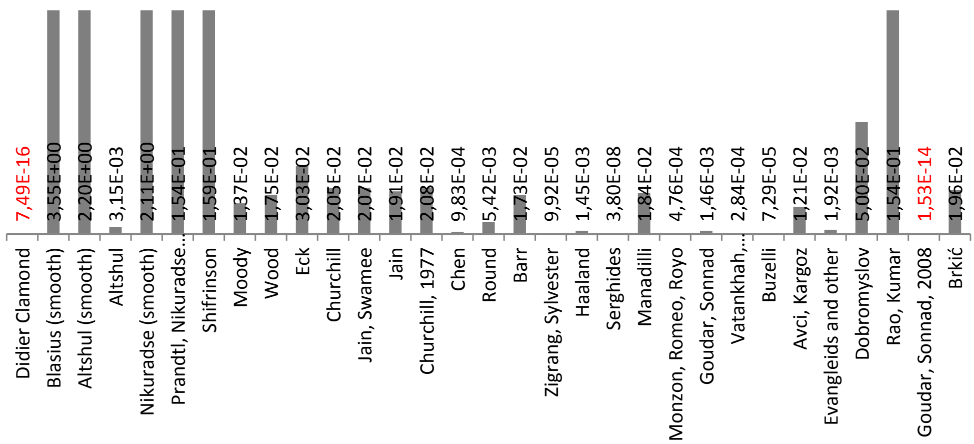

Fig. 3. Absolute mean square deviation for *k_e* / *d_int* = 0,01

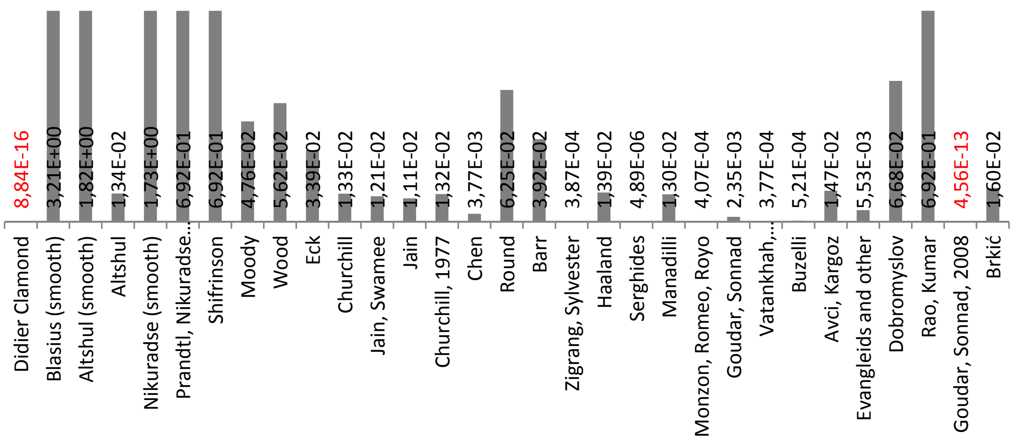

Fig. 4. Absolute mean square deviation for *k_e* / *d_int* = 0,001

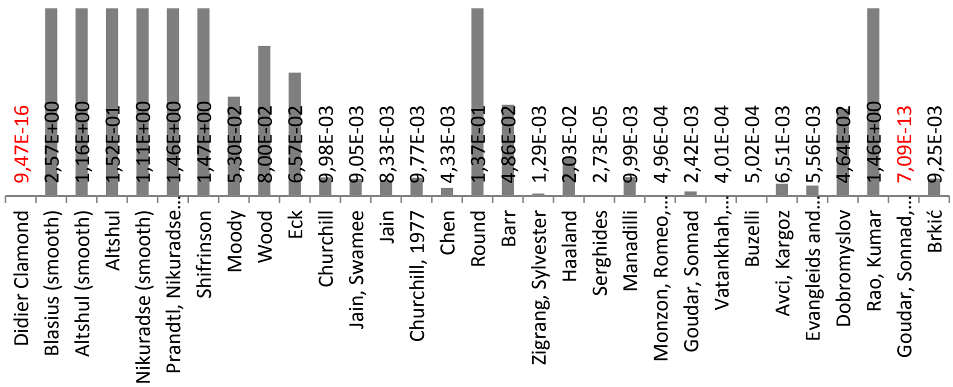

Fig. 5. Absolute mean square deviation for *k_e* / *d_int* = 0,0001

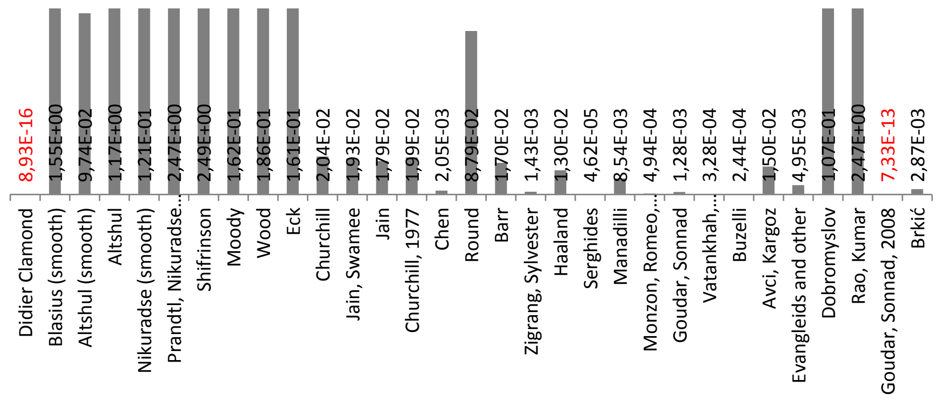

Fig. 6. Absolute mean square deviation for *k_e* / *d_int* = 0,000001

It is clear that Clamond method gives highest accuracy for all ranges of *k_e* / *d_int* . Second place goes to method of Goudar and Sonnad (2008), in the smooth pipes zone it gives almost identical accuracy, and for the rest of turbulent flow its absolute mean square deviation is 3 degrees higher. It should be noted that both methods provide much better accuracy than rest of researched functions.

Relative CPU time was also compared. Code for SciLab was written for all functions and the required computational time was measured using *timer()* function. Figure 7 shows bar-plot with results expressed in percents.

Results, obtained from Clamond method, were treated as the most accurate, and other results were compared to them afterwards. Relative deviation $abs\left(\frac{\lambda_i-\lambda_{Clamond}}{\lambda_{Clamond}}\right)\times 100\,\%$ is shown on series of plots on figures 8-22. Five series of calculation were made for different $k_e/\,d_{int}$: 0,000001; 0,0001; 0,001; 0,01; 0,05. To provide a better overview and innerview of results each of the plots is introduced in three scales – fullscale and zoomed (with relative deviation axis upper limit set to 10% and to 1%).

These plots (figures 8-22) provide an interesting insight on behavior of different equations for hydraulic friction factor, but still does not give a clear criteria to consider accuracy. That criteria would be mean square deviation of given results from ideal (which is Clamond solution in our case). Futher calculations were carried out and results are shown on bar-plots in figures 23-27.

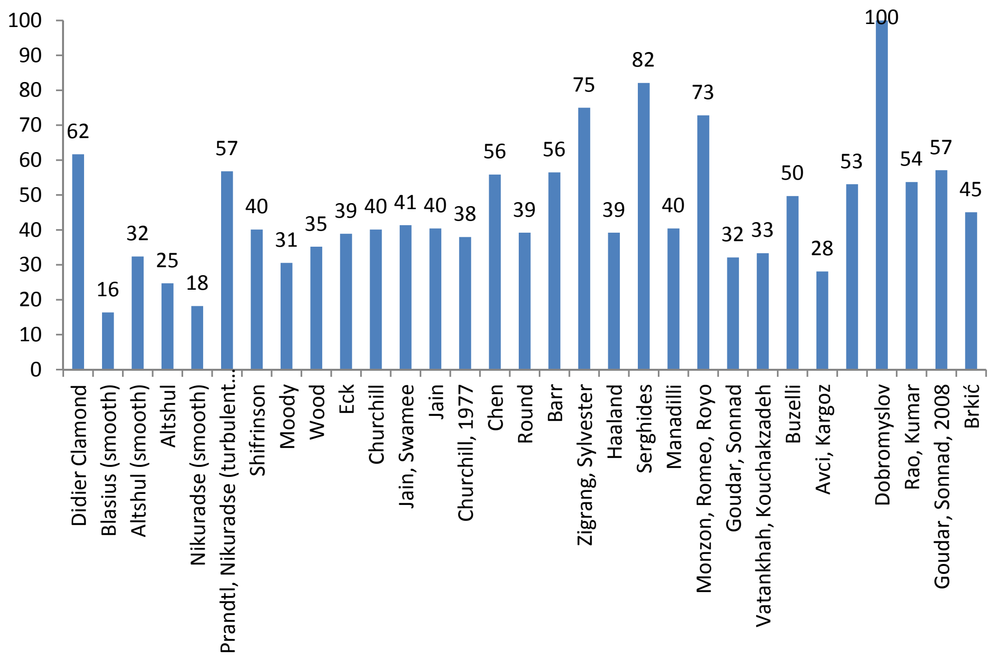

Fig. 7. Relative CPU time to compute friction factor

One way to describe λ in critical zone (fig. 1) is to build cubic interpolation function. There is widely adopted cubic interpolation developed by Dunlop. He took Poiseuille equation for laminar flow and Swamee-and-Jain equation for turbulent flow as boundary conditions.

In order to provide smooth transition from laminar regime to turbulent using more accurate solution of Colebrook-White equation given by Clamond we propose use of general cubic interpolation polynomial, which allows setting any functions as boundary conditions..

General cubic interpolation polynomial is given as

$$f_{cub}(x) = a(x - x_1)^3 + b(x - x_1)^2 + c(x - x_1) + d = 0 \tag{81}$$

We need to solve the following system of equations to find coefficients *a, b, c, d*.

$$\left.\begin{array}{r} f_{cub}(x_1) = f_1(x_1) \\ f_{cub}(x_2) = f_2(x_2) \\ f'_{cub}(x_1) = f_1'(x_1) \\ f'_{cub}(x_2) = f_2'(x_2) \end{array}\right\} \tag{82}$$

Solving system of equations (82) for *a, b, c, d* gives:

$$a = -\frac{\left(2\big(f_2(x_2) - f_1(x_1)\big) - \big(f_2'(x_2) + f_1'(x_1)\big)(x_2 - x_1)\right)}{(x_2 - x_1)^3} \tag{83}$$

$$b = \frac{\left(3\big(f_2(x_2) - f_1(x_1)\big) - \big(f_2'(x_2) + 2f_1'(x_1)\big)(x_2 - x_1)\right)}{(x_2 - x_1)^2} \tag{84}$$

$$c = f_1'(x_1) \tag{85}$$

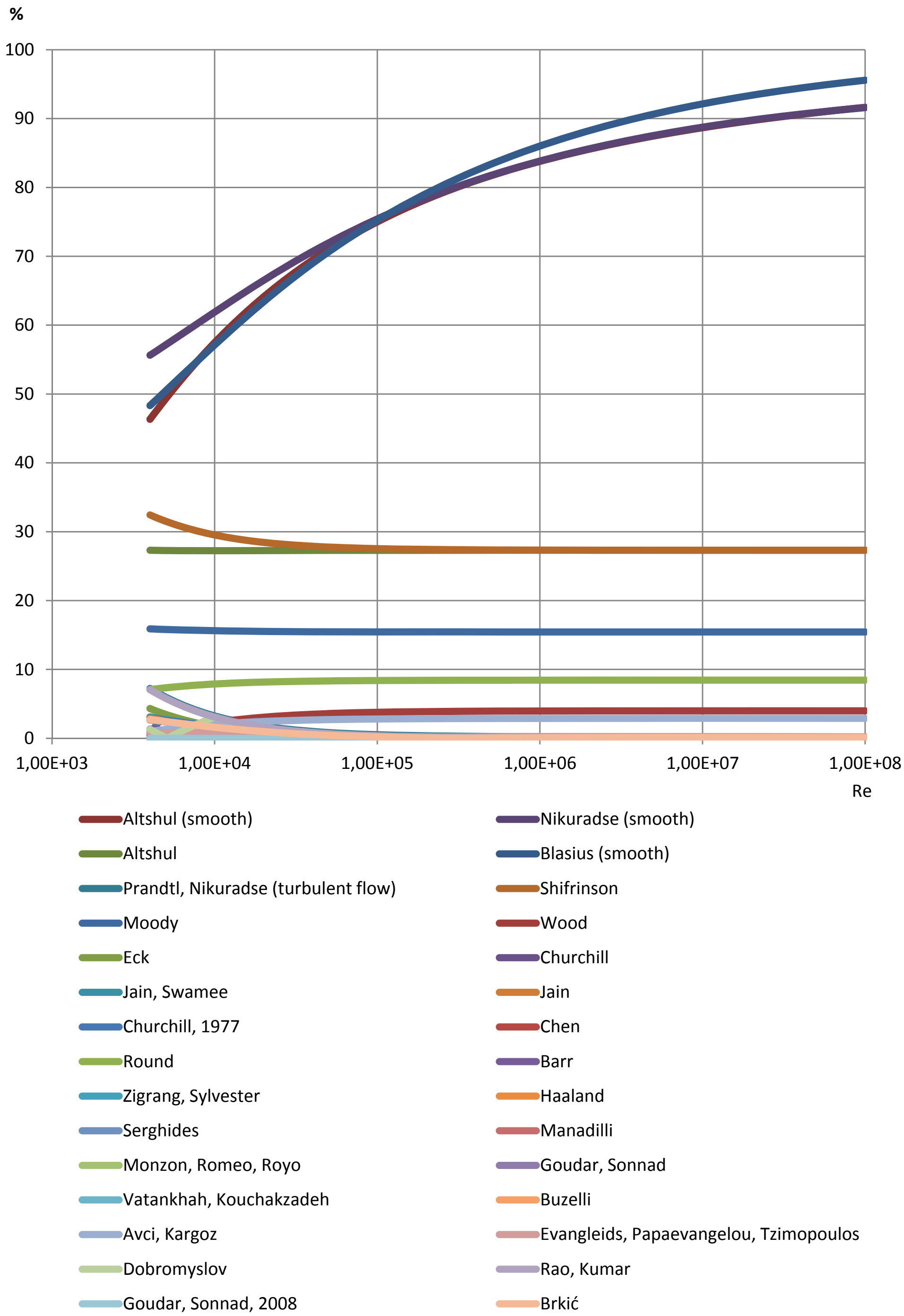

Fig. 8. Relative deviation for *k_e / d_int* = 0,05

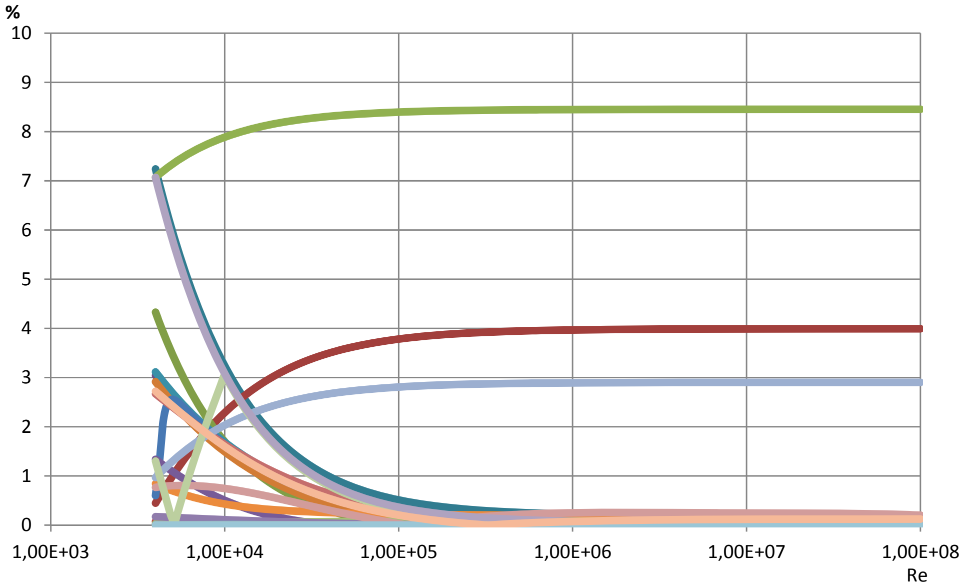

Fig. 9. Relative deviation for *k_e / d_int* = 0,05 (upper limit set to 10 %)

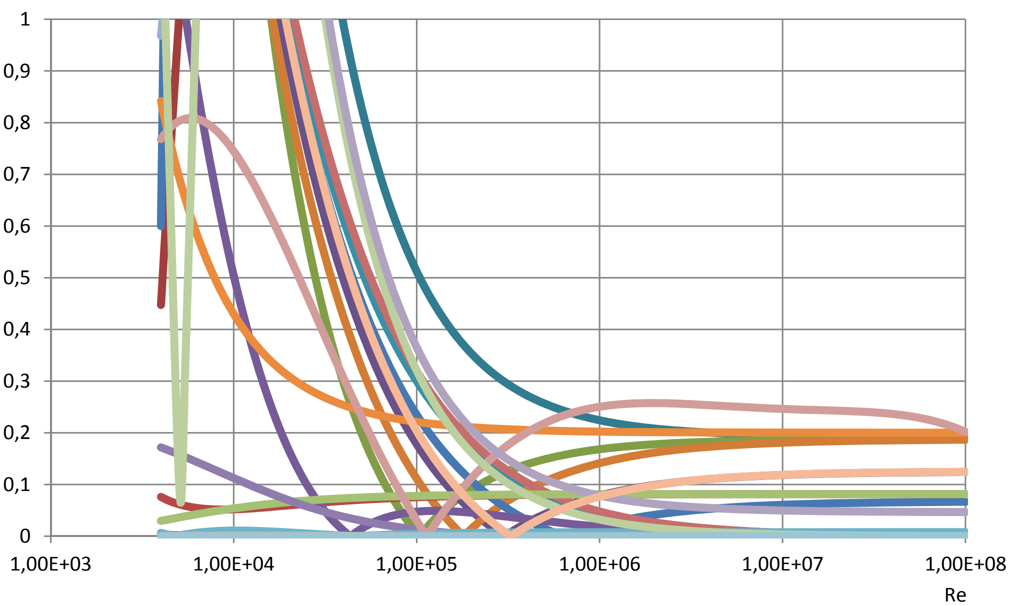

Fig. 10. Relative deviation for *k_e / d_int* = 0,05 (upper limit set to 1 %)

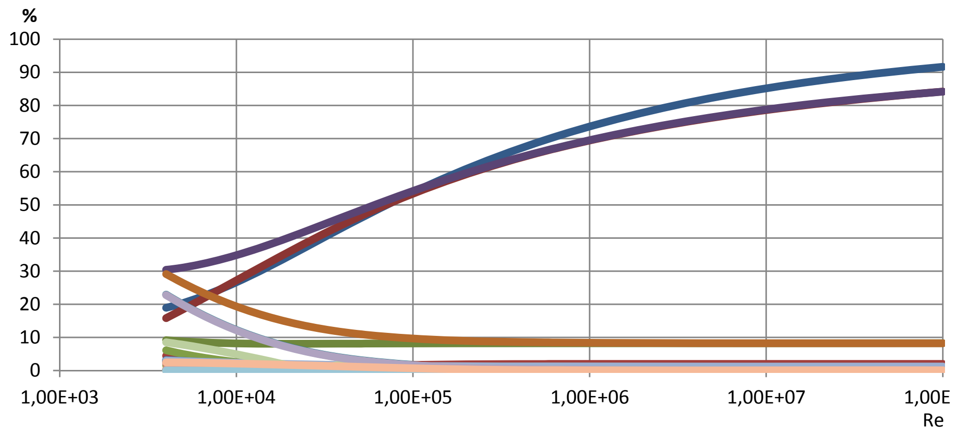

Fig. 11. Relative deviation for *k_e / d_int* = 0,01

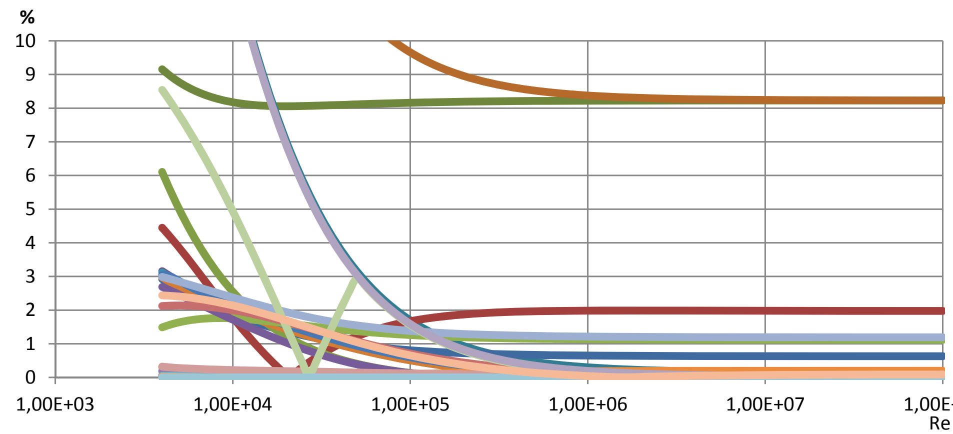

Fig. 12. Relative deviation for *k_e / d_int* = 0,01 (upper limit set to 10 %)

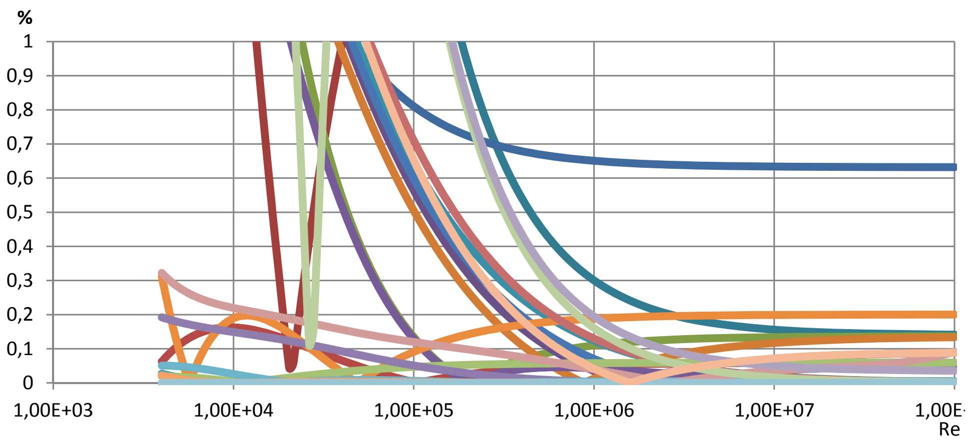

Fig. 13. Relative deviation for *k_e / d_int* = 0,01 (upper limit set to 1 %)

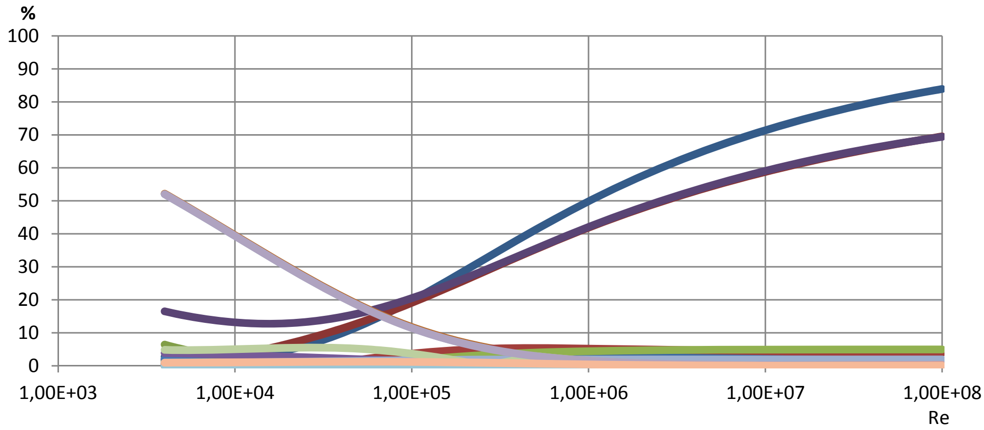

Fig. 14. Relative deviation for *k_e / d_int* = 0,001

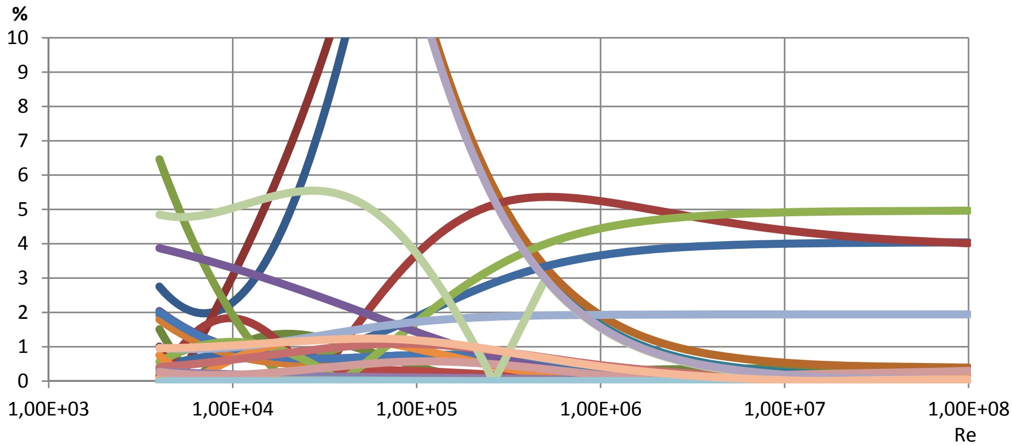

Fig. 15. Relative deviation for *k_e / d_int* = 0,001 (upper limit set to 10 %)

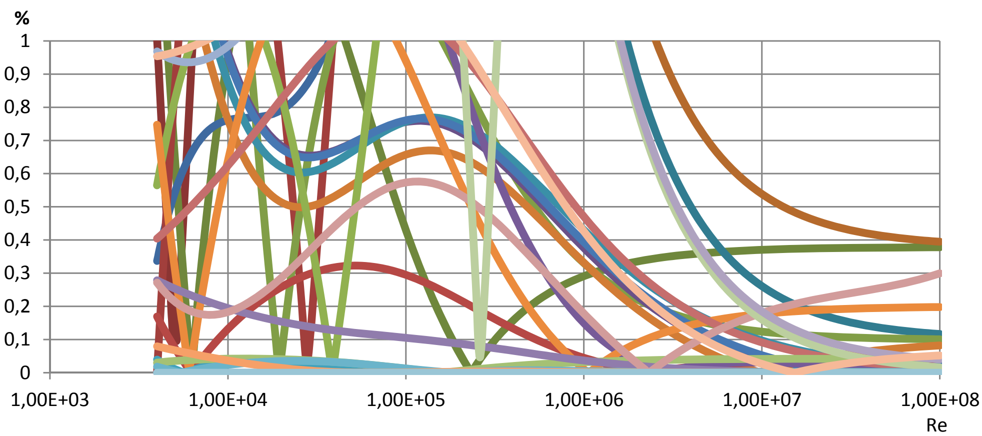

Fig. 16. Relative deviation for *k_e / d_int* = 0,001 (upper limit set to 1 %)

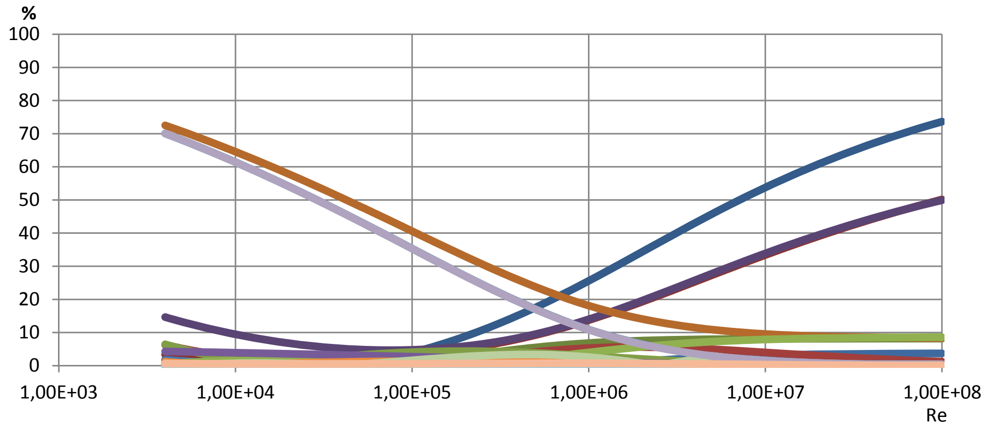

Fig. 17. Relative deviation for *k_e / d_int* = 0,0001

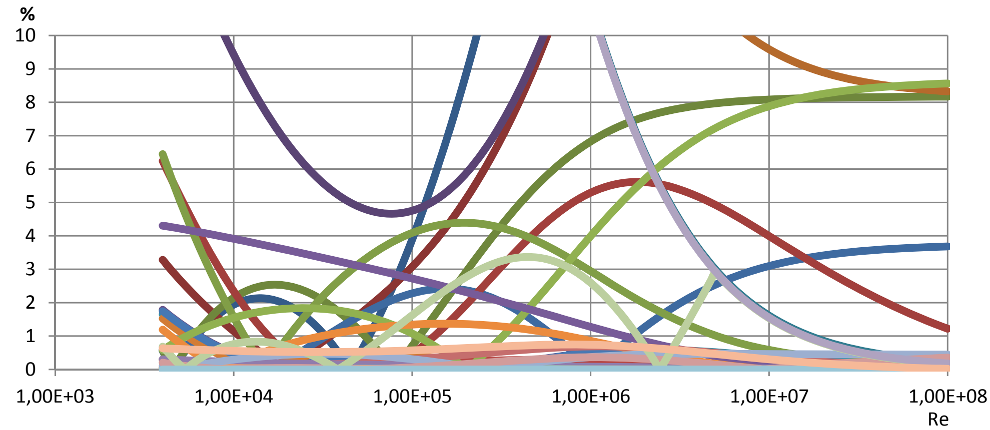

Fig. 18. Relative deviation for *k_e / d_int* = 0,0001 (upper limit set to 10 %)

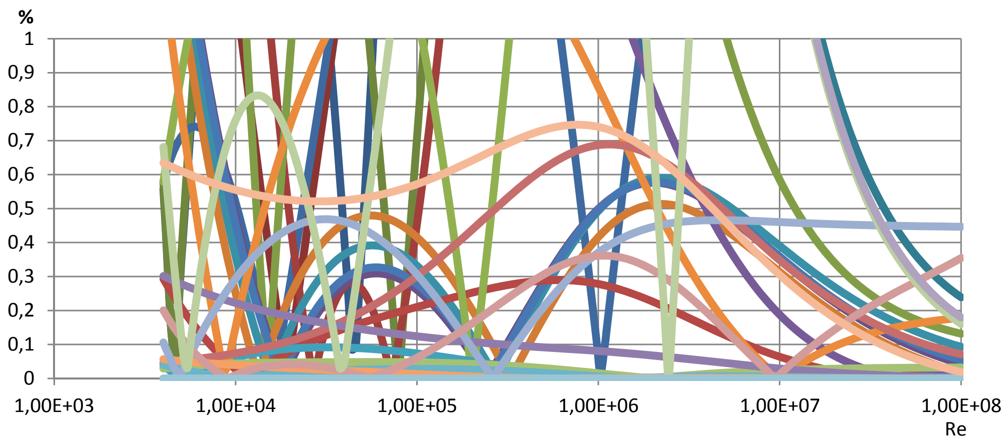

Fig. 19. Relative deviation for *k_e / d_int* = 0,0001 (upper limit set to 1 %)

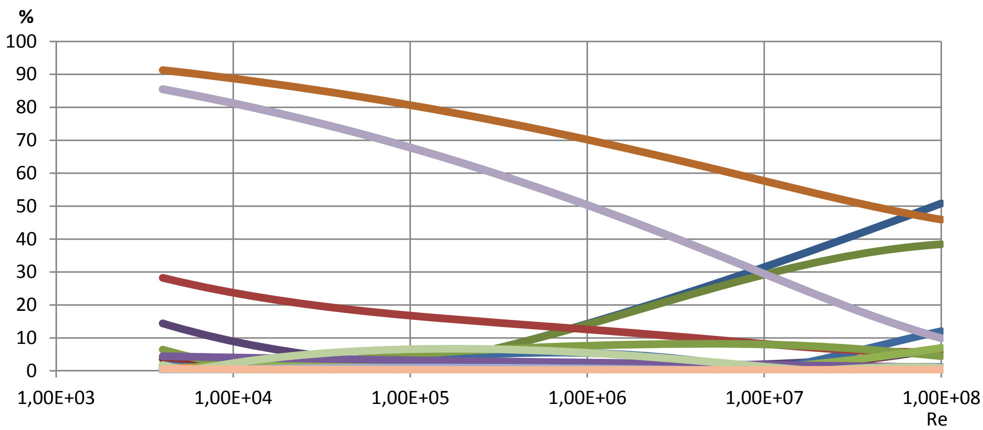

Fig. 20. Relative deviation for *k_e / d_int* = 0,000001

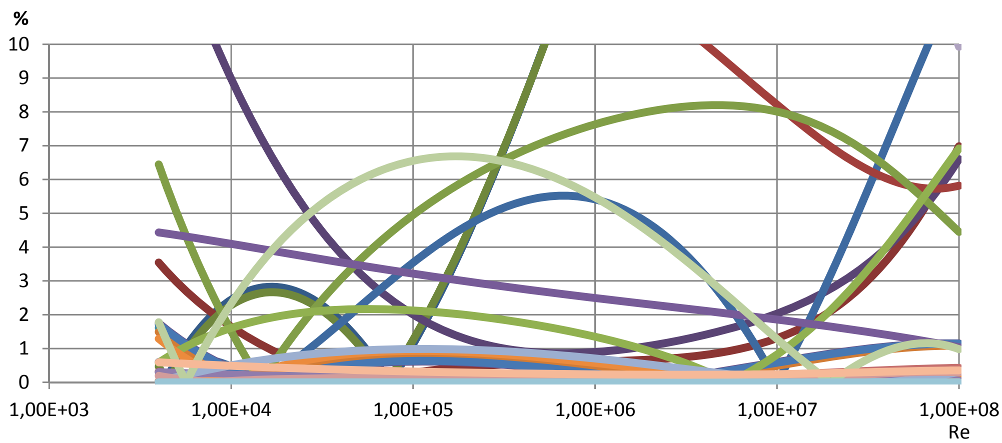

Fig. 21. Relative deviation for *k_e / d_int* = 0,000001 (upper limit set to 10 %)

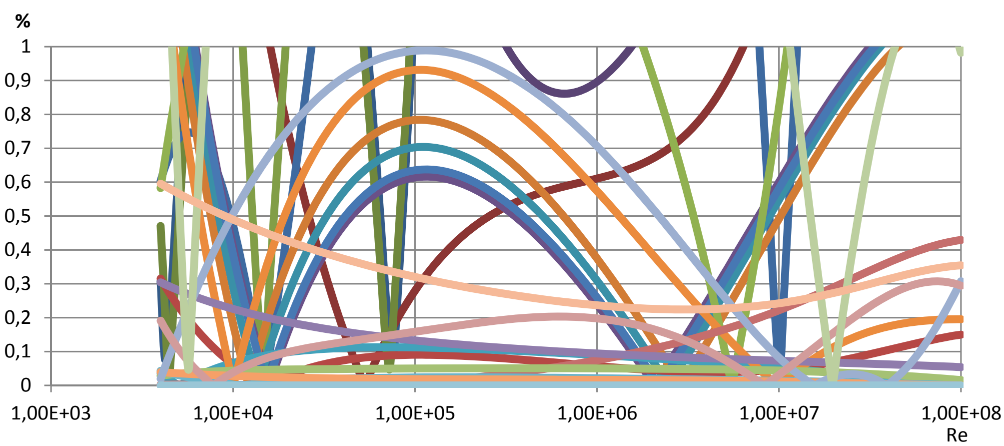

Fig. 22. Relative deviation for *k_e / d_int* = 0,000001 (upper limit set to 1 %)

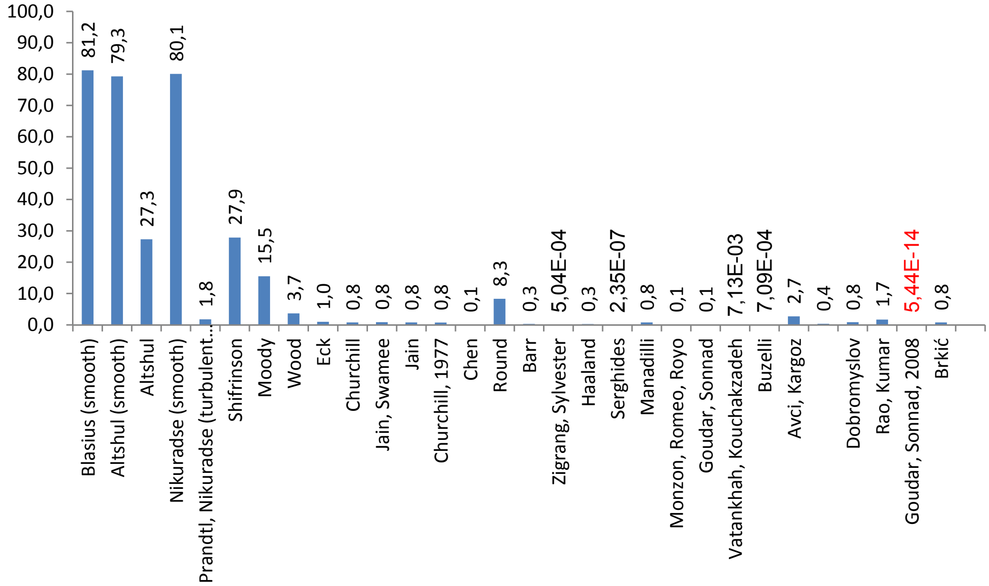

Fig. 23. Mean square deviation for *k_e* / *d_int* = 0,05

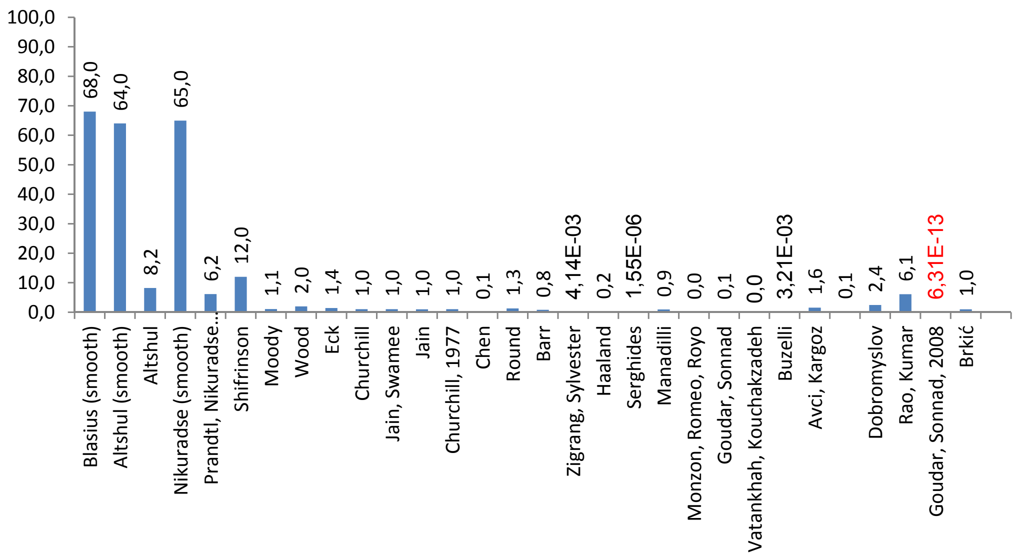

Fig. 24. Mean square deviation for *k_e* / *d_int* = 0,01

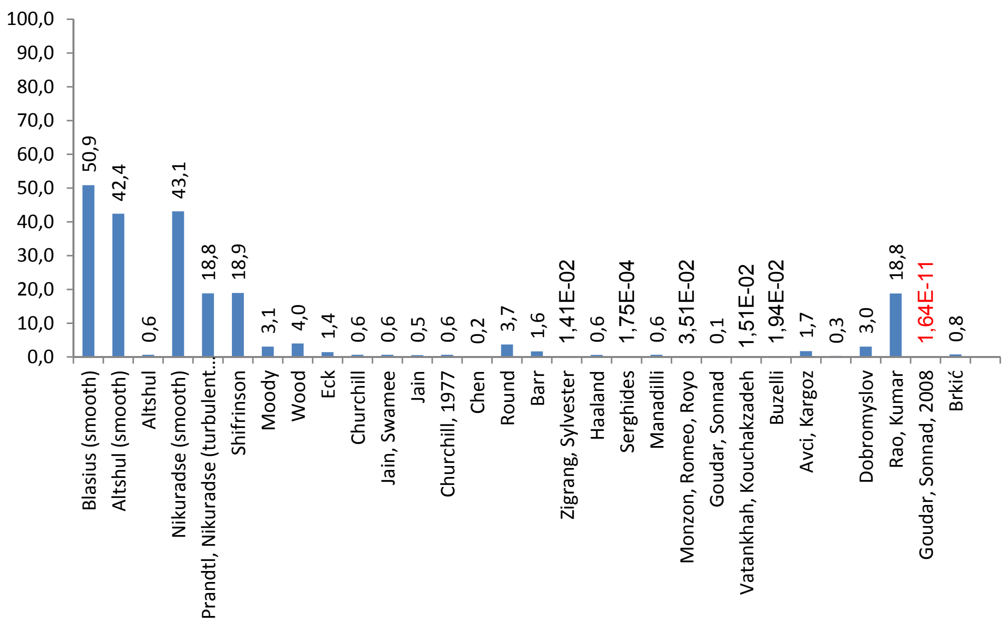

Fig. 25. Mean square deviation for *k_e* / *d_int* = 0,001

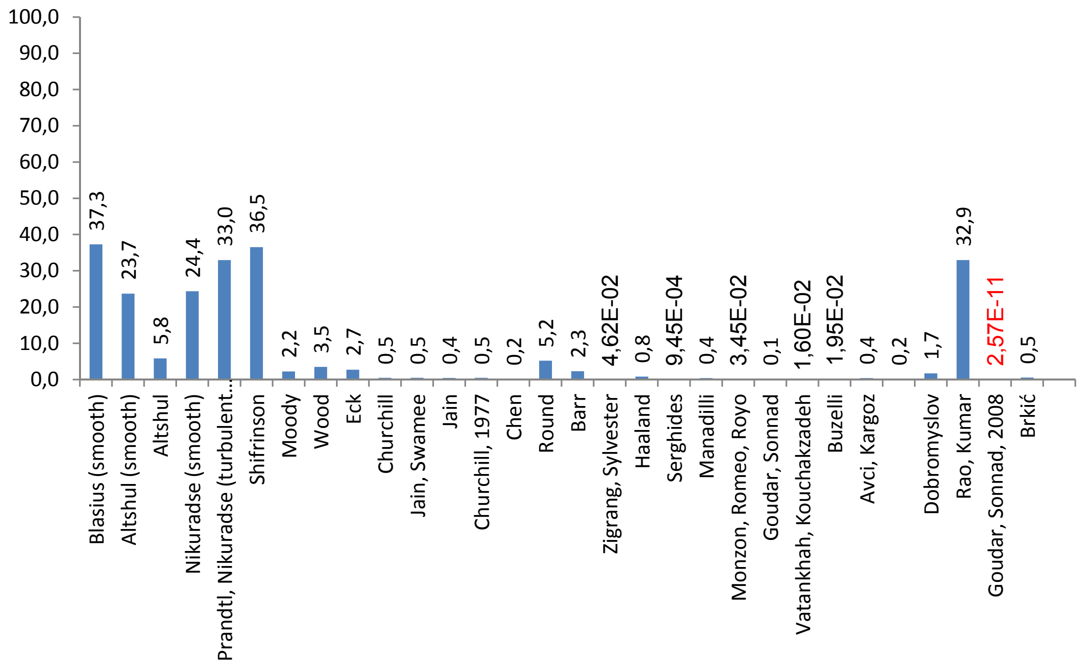

Fig. 26. Mean square deviation for *k_e* / *d_int* = 0,0001

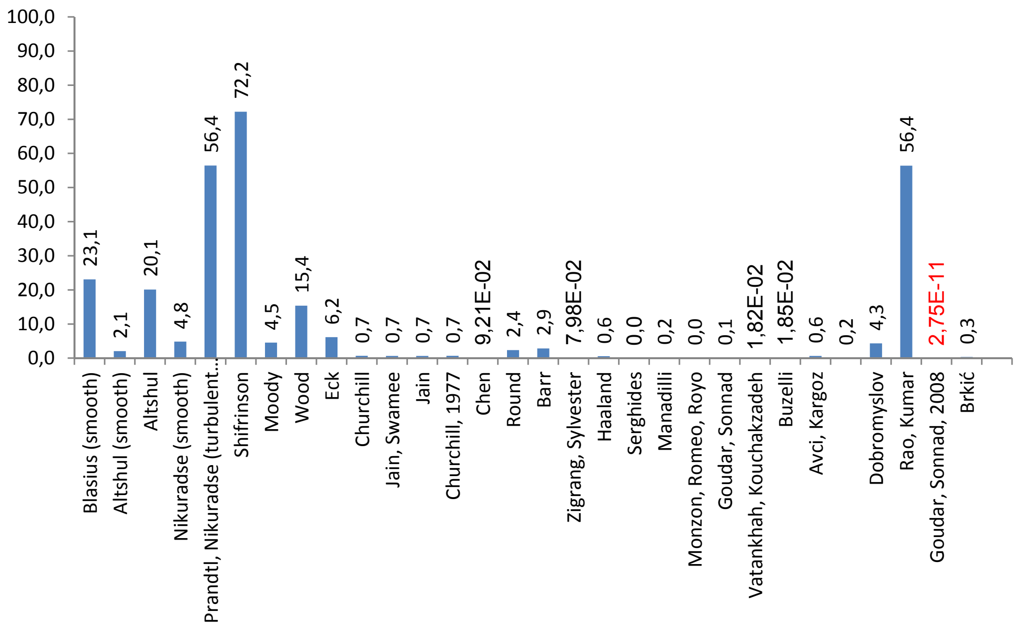

Fig. 27. Mean square deviation for *k_e / d_int* = 0,000001

$$d = f_1(x_1) \tag{86}$$

It is widely accepted in hydraulic calculations that critical zone lays in 2000 < *Re* < 4000, which is why $x_1 = 2000$, $x_2 = 4000$.

Differential can be computed numerically

$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x}. \tag{87}$$

Conclusion

Results of comparative analysis provide engineers and software developers a clear choice of method to choose based on accuracy (figures 23-27) and computational time (fig. 7)

Method of Clamond to solve Colebrook-White equations clearly sets aside from other methods because of its constant highly accurate results for all ranges of Reynolds number and *k_e / d_int*.

We propose easy to use algorithm of cubic interpolation for critical zone, which provides smooth transition and allows using any chosen functions as boundary.

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Определение коэффициента гидравлического трения в трубопроводных системах

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Выполнен сравнительный анализ многих известных формул для определения коэффициента гидравлического трения в трубах с точки зрения точности и скорости расчета. Для обеспечения плавного перехода от ламинарного режима к переходному в критической зоне предложен алгоритм кубической интерполяции общего вида.

Ключевые слова: коэффициент гидравлического трения, критическая зона, трубопроводные системы, интерполяция.